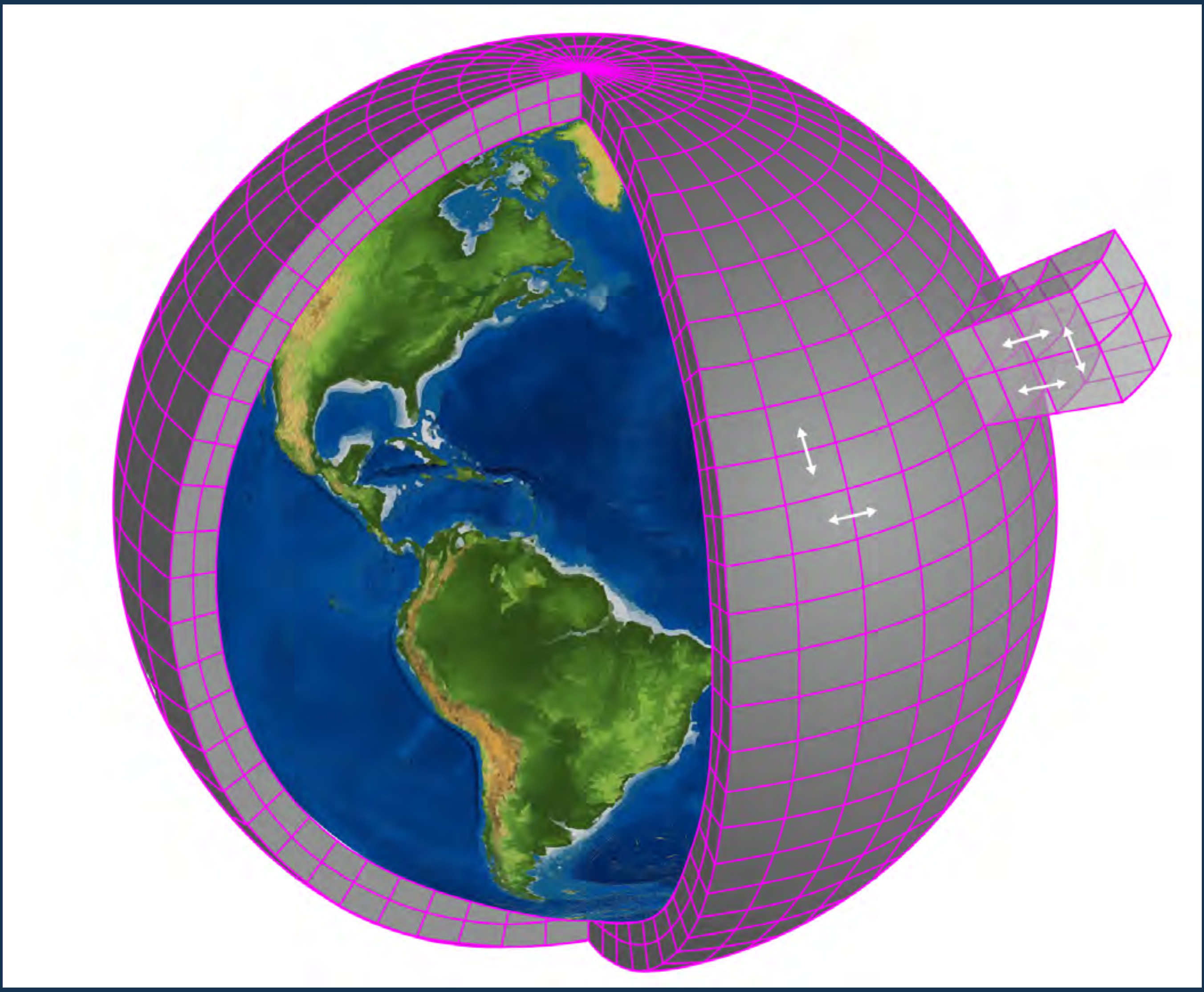
LONDON MATHEMATICAL SOCIETY
EST. 1865

NEWSLETTER

Issue: 511 - May 2024



MATHEMATICS OF CLIMATE MODELS

NOETHERIAN EXTENSIONS OF COMMUTATIVE RINGS

NOTES OF A NUMERICAL ANALYST

EDITORIAL OFFICE

London Mathematical Society
De Morgan House
57–58 Russell Square
London WC1B 4HS
newsletter@lms.ac.uk

Charity registration number: 252660

COVER IMAGE

Cover image: Figure 1 from Emily Potter's article 'The Mathematics of Climate Models' (page 14).

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email images@lms.ac.uk for details.

COPYRIGHT NOTICE

ISSN: 2516-3841 (Print)
ISSN: 2516-385X (Online)
DOI: 10.1112/NLMS

NEWSLETTER WEBSITE

The Newsletter is freely available electronically at lms.ac.uk/publications/lms-newsletter.

MEMBERSHIP

Joining the LMS is a straightforward process. For membership details see lms.ac.uk/membership.

SUBMISSIONS

The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see lms.ac.uk/publications/advertise-in-the-lms-newsletter.

CONTENTS

LMS NEWS

Maths Summit and Manifesto

Delegates at the event in the Science Museum

The mathematical community, together with politicians, policymakers, funders and representatives from industry, gathered at the Science Museum in London on 12 March 2024 for the Maths Summit. This was a one-off event to explore how the mathematical sciences can better support research, innovation and prosperity in the United Kingdom. Organised by the Protect Pure Maths campaign and supported by the Council for Mathematical Sciences, the LMS and the four other learned mathematical societies, the summit was an unprecedented event in bring the community together ahead of the forthcoming UK General Election.

Overall, the event aimed to build positive support around the next steps for the mathematical sciences and to showcase and celebrate the importance of the mathematical sciences to the whole of the UK. Through keynote speakers, panels, exploratory discussions and exhibitions the event was able to consider how the contribution of the UK mathematical community can be maximised to meet socio-economic challenges, and how the sector can be supported more effectively by politicians, policymakers and businesses. The Summit included sessions on the value of mathematical sciences to society and policy-making, explore key issues and risks around research funding and teaching provision, and build support for policies and solutions for the next Government. Speakers included the Secretary of State for Science, Industry and Technology, the Director of GCHQ, the Executive Chair of EPSRC and the National Statistician as well as eminent mathematicians and senior industry figures.

Further details can be found at: themathssummit.co.uk.

The Summit was also used to launch the 'Maths Manifesto', a vision statement produced by the Council for the Mathematical Sciences and the Academy for the Mathematical Sciences. The purpose of the Manifesto is to outline the community's vision for how mathematics can achieve an even greater impact on the UK and to set out priorities for the next Parliament and beyond in this Westminster election year. It calls upon political parties, and all those with an interest, to play their part in creating a future where mathematics can play a full role in supporting nationwide prosperity and policy making as well as making a difference to individual lives. LMS members are encouraged to use the Manifesto to shape discussions they may be involved in and to bring it to the attention of their local MPs.

The Maths Manifesto can be found on the LMS website: lms.ac.uk/news/maths-manifesto.

Catherine Hobbs
LMS Vice-President

10ECM Bid

The LMS and the Institute of Mathematics and its Applications have submitted a joint bid to host ECM10 in London in 2028. The joint bid highlights the outstanding and diverse mathematics community in the UK and proposes hosting the congress at the ExCeL conference centre in East London. A local organising committee has been established with Richard Thomas (Imperial) and Helen Wilson (UCL) as co-Chairs. The bid process started in June 2023 and was led by past Presidents Ulrike Tillmann (LMS) and Paul Glendinning (IMA). Current Presidents Jens Marklof (LMS) and Hannah Fry (IMA) now lead the bid on behalf of the two learned societies. In July 2024, the bid will be presented by Ulrike Tillmann to the European Mathematical Society Council in Granada, Spain. The voting will take place the same weekend and the results will be known at the start of 9ECM in Seville. We wish the LMS/IMA bid team the best of luck!

Jennifer Gunn
LMS Head of Society Business

Publication Ethics in an Era of Threats to Research Integrity

Since the development of the first academic journal in the 17th century, the ecosystem of journal publishing has operated under the basic assumption that everyone participates in the system in good faith. Sadly, ensuring research integrity and ethical practice has become one of the biggest challenges for academic journals in recent years.

Widespread fraud perpetrated by paper mills — commercial organisations which produce fake research articles and sell authorship to those willing to pay — has resulted in some journal publishers retracting hundreds of fraudulent articles which had managed to pass their review process undetected. And the dawn of generative AI (GenAI) tools, like ChatGPT, means that fabricating a fraudulent but plausible looking research paper is easier than ever before.

GenAI also raises new questions for the publishing world. What is the nature of authorship, and could an AI qualify as an author? Is it possible to use GenAI and still maintain confidentiality of the peer-review process? There are also ongoing debates (and legal action) about whether the training of GenAI tools on copyrighted material constitutes fair use or is a violation of intellectual property rights.

As an internationally trusted publisher of peer-reviewed mathematical research, the Society keeps up to date with the latest developments in these areas to ensure we maintain the highest standards of research integrity and publication ethics.

The intensive and rigorous peer-review process employed by the Society's journals means that we are resilient in the face of paper mill fraud. A recent scan of the Society's archive of journal articles with 'Papermill Alarm' — the market leading tool for detecting fraudulent articles — showed no evidence that the Society's journals have ever been compromised by paper mills.

We are also proud to announce that the Society has launched a new Ethical Policy for Journals. This new policy tightens up the Society's existing position on many aspects of publication ethics, as well as addressing newer issues like paper mills and GenAI. In order to be as accessible as possible, the policy now has clear and distinct codes of ethics for authors, reviewers and editors. And, in line with the Society's commitment to equity, diversity and inclusion, the ethical policy now enshrines these principles as a core ethical issue for journals.

The new policy can be accessed via the Society's website at lms.ac.uk/publications.

The Society's continuing work in this area means that authors, reviewers and editors know that they will participate in a fair and ethical peer-review process with the LMS journals, and that readers can be confident in the integrity of the research that we publish.

Simon Buckmaster
LMS Head of Academic Publications

Cecil King Travel Scholarship Recipients

Alp Müyesser and Laura Johnson

The LMS and the Cecil King Memorial Foundation are pleased to announce that two Cecil King Travel Scholarships 2024 were awarded to early career mathematicians Laura Johnson and to Alp Müyesser.

Established in honour of British newspaper proprietor and businessman Cecil Harmsworth King, these scholarships aim to support early career individuals who demonstrate exceptional academic potential in the field of mathematics. The funding allows young mathematicians to undertake a study or research visit abroad for a period of three months.

About the 2024 Award Recipients

Laura Johnson is a PhD student at the University of St. Andrews. She is interested in combinatorial objects that have applications in information security. Currently, the majority of Laura's research centres around finding new constructions of difference families, which are collections of combinatorial structures with applications in both cryptography and coding theory. Further to this, she is also working to develop new cyclotomic techniques in finite fields and interested in combinatorial designs and finite geometry. Laura will visit Anita Pasotti at the University of Brescia, Italy to conduct a research project exploiting connections between Heffter arrays and difference families to obtain new authentication code constructions.

Alp Müyesser is a PhD Candidate at University College London who is interested in extremal and probabilistic combinatorics. Specifically, he studies the existence of large-scale substructures in graphs and hypergraphs. Recently, Alp has been interested in pseudorandom graphs, which are graphs where the statistics of the connection patterns resemble that of a graph generated randomly. There are several old open problems concerning how pseudorandom a graph needs to become before it necessarily contains a certain large-scale substructure. The motivation for such problems come from Cayley graphs, which often display remarkable pseudo randomness properties. In addition, Alp is also interested in Latin squares, additive combinatorics, and Ramsey theory.

The Cecil King Travel Scholarship will fund Alp's research visit to Professor Matija Bucic at Princeton University, USA. They will conduct research in intersection of discrete mathematics, group theory, and computer science, proposing to further the theory of "robust sublinear expanders", a thriving area in combinatorics with deep connections to algebra.

The London Mathematical Society and the Cecil King Memorial Foundation believe that the scholarship program offers early career mathematicians various opportunities in form of mentorship, professional development, networking and progress of their academic careers.

If you would like to find more about Cecil King Travel Scholarships and how to apply, please visit lms.ac.uk/prizes/cecil-king-travel-scholarship.

The application deadline for the 2025 programme is 15 November 2024. The application form will be available on the LMS website in September. Shortlisted applicants will be invited to interview during which they will be expected to make a short presentation on their proposal. Interviews will take place in January 2025.

Valeriya Kolesnykova
LMS Accounts, Membership & Fellowship Assistant

LMS Early Career Fellowship Recipients

The Society is thrilled to announce the recipients of the highly competitive 2023-24 LMS Early Career Fellowships. This funding provides support for talented UK mathematicians who have recently completed their PhD and are actively looking to secure their next postdoctoral position. During this transition period, a stipend of £1,552 per month is awarded for a research visit between 3 to 6 months, along with £800 for relocation or research collaboration visits. LMS Early Career Fellowships not only provide financial support for research and career development, but also help to foster collaboration and promote the advancement of mathematics as a discipline.

This year, the Early Career Research Committee considered 42 applications and awarded 8 Fellowships. These awards will support the researchers at universities both in the UK and overseas. A full list of the current Fellows is available on the Society's website: lms.ac.uk/grants/lms-early-career-fellowships#Current.

Applications for the next round will open later in 2024, with a deadline of 14 January 2025 to support Fellowships starting from April 2025. For full details, see lms.ac.uk/grants/lms-early-career-fellowships or direct your enquiries to fellowships@lms.ac.uk.

LMS Early Career Fellowships are funded with support from the Heilbronn Institute for Mathematical Research (HIMR) and UKRI/EPSRC.

Valeriya Kolesnykova
LMS Accounts, Membership & Fellowship Assistant

Forthcoming LMS Events

The following events will take place in forthcoming months:

LMS Popular Lecture 2024 (Speaker: Sarah Hart) 9 May, Birmingham (tinyurl.com/wnarjs5z)

LMS Maths Communication Workshop (with Ben Sparks and Katie Steckles:) 21 May, online (tinyurl.com/3eenyha5)

LMS/Gresham Lecture 2024 (Speaker: Oliver Johnson): 22 May, London and online (tinyurl.com/bdcs4j92)

LMS Education Day: Mathematics, AI and DigiTech Futures: 22 May, London and online (tinyurl.com/47x9wypj)

LMS Society Meeting at the BMC 2024 (Speaker: Corinna Ulcigrai): 19 June, Manchester (tinyurl.com/5xcen67d)

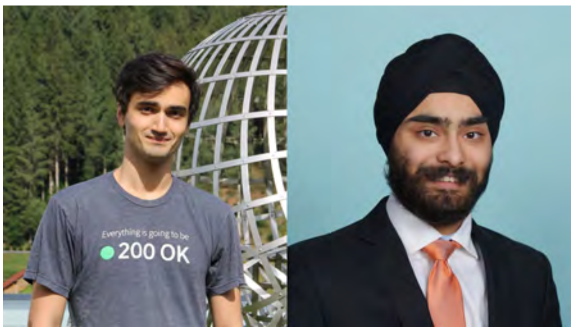
LMS General Meeting and Celebration of Kelvin's 200th Anniversary (in partnership with BSHM and University of Glasgow): 28 June, London and online (tinyurl.com/35vzsuk4)

LMS Invited Lecture Series 2024 (Invited Lecturer: Dan Abramovich): 1-5 July, London (tinyurl.com/bddtvtda)

A full listing of upcoming LMS events can be found on page 37.

OTHER NEWS

Clay Research Fellows 2024



Ishan Levy and Mehtaab Sawhney

The Clay Mathematics Institute is pleased to announce that Ishan Levy and Mehtaab Sawhney have been awarded Clay Research Fellowships.

Ishan Levy will receive his PhD from the Massachusetts Institute of Technology in 2024, under the supervision of Michael Hopkins. Levy is known for his deep and ingenious contributions to homotopy theory. His new techniques in algebraic K-theory have led to solutions of many old problems. In joint work with Burklund he established the rational convergence of the "Waldhausen Tower" interpolating between the K-theory of the integers and one of the most important moduli spaces in the study of high dimensional manifolds, A(pt). He is most renowned for his work Ravenel's "Telescope Conjecture". In the late 1970s Ravenel made a series of deep conjectures outlining a rich conceptual vision of stable homotopy. By the mid-1980s all but the Telescope Conjecture had been proved. For over 40 years this remained the most important problem in this part of homotopy theory. Levy's methods in K-theory led him, Burklund, Hahn and Schlank, to the construction of counterexamples, and, in joint work with Burklund, Carmeli, Hahn, Schlank and Yanovski, to an estimate of the growth rate of the stable homotopy groups of spheres that was completely untouchable by previous methods. Ishan Levy has been appointed as a Clay Research Fellow for five years beginning 1 July 2024.

Mehtaab Sawhney will receive his PhD from the Massachusetts Institute of Technology in 2024, under the supervision of Yufei Zhao. While still a graduate student, Sawhney has achieved a stunning number of breakthroughs on fundamental problems across extremal combinatorics, probability theory, and theoretical computer science. He is a highly collaborative researcher whose partnership with Ashwin Sah has been particularly fruitful. His remarkable body of work has already transformed swathes of combinatorics. For example, working with Kwan, Sah and Simkin, he proved a 1973 conjecture of Erds on the existence of high-girth Steiner triple systems; with Keevash and Sah he established the existence of subspace designs; with Jain and Sah he established sharp estimates for the singularity probability in a wide class of discrete random matrices; with Sah and Sahasrabudhe he showed the existence of the spectral distribution of sparse directed Erds–Rényi graphs; and with Kwan, Sah and Sauermann, he developed highly novel tools in anti-concentration in order to prove the Erdős–McKay conjecture concerning edge statistics in Ramsey graphs. Mehtaab Sawhney has been appointed as a Clay Research Fellow for five years beginning 1 July 2024. Clay Research Fellowships are awarded on the basis of the exceptional quality of candidates' research and their promise to become mathematical leaders. For more information visit claymath.org.

International Mathematics Competition for University Students

The 31st IMC will be held 5-11 August 2024 in Blagoevgrad, Bulgaria. It is organised by University College London and hosted by the American University in Bulgaria. Universities are invited to send several students and one teacher as Team Leader. Individual students without Team Leaders are welcome. The competition is planned for students just completing their first, second, third, or fourth years of university education and will consist of two sessions of five hours each. Problems will be from the fields of Algebra, Analysis (Real and Complex), Geometry and Combinatorics. The maximum age of participants is normally 23 years of age at the time the IMC, although exceptions can be made. The working language will be English.

The IMC in Blagoevgrad is a residential competition, and all student participants are required to stay in the accommodation provided by the hosts. It aims to provide a friendly, comfortable and secure environment for university mathematics students to enjoy mathematics with their peers from all around the world, to broaden their world perspective and to be inspired to set mathematical goals for themselves that might not have been previously imaginable or thought possible. Past participants have gone on to distinguished careers in mathematics; for example, Field's Medallists Cauchar Birkar and Maryna Viazovska both participated in IMCs.

Over the past 30 Competitions the IMC has had participants from over 200 institutions from over 50 countries. For further information and online registration visit the website at www.imc-math.org.uk. Further details may be obtained from Professor John Jayne (j.jayne@ucl.ac.uk).

Mathematics for Humanity Activities Worldwide

ICMS has announced it is now able to fund international activities as part of its Mathematics for Humanity programme. This has been made possible by a generous donation from Alex Gerko, Founder of XTX Markets. Applicants are invited to submit proposals belonging to one of the three themes for a satellite event at any location in the world that is optimal for impact, accessibility, and sustainability. As part of this effort, we expect to be able to fund up to ten global workshops for humanity with a maximum budget of £35,000 each. The usual guidelines and application forms can be used for this programme.

However, in addition to the scientific case, organisational outline, and a strong argument for benefit to humanity, the proposers should provide evidence that:

(1) The location serves well the goals of impact, accessibility, and sustainability;

(2) The chosen venue is well-equipped for event delivery;

(3) There is an efficient and reliable system for transfer and administration of funds from the UK.

It is recommended very strongly that the plans be supported by a local university or institute with international standing, which should be specified on the application. As examples, the Vietnam Institute for Advanced Study in Mathematics has expressed a willingness to be hosts and provide help with local administration of the funds from the ICMS Director for events with a focus on Southeast Asia, whilst the African Institute for Mathematical Sciences has agreed to host events centred in Africa. Write to the director Minhyong Kim (minhyong.kim@ icms.org.uk) for queries. Closing date is 1 June 2024.

MATHEMATICS POLICY DIGEST

Maths degrees need a new formula — but not greater specialisation

Ahead of the Maths Summit on 12 March, an article by LMS President Jens Marklof was published on the *Times Higher Education* blog shedding light on challenges for mathematics departments and calling for 'a new formula for maths degrees'. In it, Prof. Marklof argued for broader mathematics degree programmes that fully embrace the changes brought about by the digital revolution, which provide pathways to the wide-ranging applications mathematics has to offer. This is an essential step to making mathematics programmes attractive to a wide and diverse pool of students.

Read the full article on the LMS website at bit.ly/4aIrTEA.

£1 billion doctoral training investment announced

On 12 March the Science, Innovation and Technology Secretary Michelle Donelan announced a £1 billion investment in 65 EPSRC centres for doctoral training (CDTs). The CDTs will support leading research in areas such as artificial intelligence, quantum technologies, semiconductors, telecoms and engineering biology.

See details at bit.ly/4asA5ZS.

Government response to National Academy consultation published

On 27 March the Government Department for Science, Innovation and Technology (DSIT) published its response to the call for evidence on a National Academy for Mathematical Sciences. Following consideration of the feedback, the objectives for the Academy were updated as follows:

- Helping the sector to speak with one voice, encouraging and respecting diverse views, developing clear and coherent positions on how to promote and enhance mathematical sciences at all levels, and across all areas of society;
- Promoting mathematical sciences in ways which support economic growth and societal benefits, including through forging links between industry and academia;
- Convening, coordinating, and assessing views and evidence from across the mathematical community so as to provide high-quality independent advice to government and society;
- Strengthening the UK's mathematical sciences sector by working constructively with existing organisations and the wider UK R&D sector, expanding and diversifying the talent pool in the UK, and forging links across academia, industry, education, government, global partners and wider civil society;
- Promoting the benefits of mathematical sciences and developing strategies to support the public understanding, trust and proficiency in, mathematical sciences;
- Championing and providing expert advice to enhance mathematical sciences education at all stages, improving mathematical literacy and boosting uptake of skills, qualifications, and careers in all parts of the UK.

DSIT set out some short- and long-term requirements for establishing the National Academy and invited a final round of feedback on these, with a deadline of 10 April. It is expected that the final objectives will be published and potential applicants to deliver the Academy will be invited to apply in spring 2024.

Read the full response at bit.ly/3TYdFtS.

Digest prepared by Katherine Wright
LMS Communications and Policy Manager

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

OPPORTUNITIES

LMS Grant Schemes

Applications are invited for the following grants to be considered by the Research Grants Committee at its June 2024 meeting. Applicants for LMS Grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands:

Application deadline: 15 May 2024

- Conferences (Scheme 1): Up to £5,500 available per grant to provide partial support for conferences.
- Visits to the UK (Scheme 2): Up to £1,500 available per grant to provide partial support for a lecturer to give lectures in at least three separate institutions.
- Research in Pairs (Scheme 4) For mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research. For those collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit.
- Research Reboot (Scheme 4) Up to £500 is available for accommodation, subsistence and travel plus an additional £500 for caring costs are available to assist UK mathematicians who have had a break due to illness, caring responsibilities, increased teaching or other factors.
- Collaborations with Developing Countries (Scheme 5): For those mathematicians inviting a collaborator to the UK, up to £3,000 is available to support a visit for collaborative research. For those going to their collaborator's institution, up to £2,000 is available.
- Research Workshop Grants (Scheme 6): up to £10,000 is available to provide support for Research Workshops.
- Mathematics in Africa Grants: up to £2,000 is available to provide partial support for mathematical activities based in Africa.

See lms.ac.uk/grants/research-grants or contact grants@lms.ac.uk for more information.

Applications for the following will be considered by the Computer Science Committee at a meeting in June 2024.

Application deadline: 4 June 2024

- Computer Science Small Grants (Scheme 7): up to £750 available per grant to support visits to undertake collaborative research at the interface of mathematics and computer science.

See details at lms.ac.uk/grants/scheme-7.

MARM Partnership Grants: Call for Expressions of Interest

Application deadline: 2 June 2024, 23:59 (BST)

The MARM (Mentoring African Research in Mathematics) Partnership Scheme awards grants of £4,000 to support partnerships between UK/European and African mathematics departments. Four grants are awarded in total. These will be for a one-year partnership initially, with the possibility of extension for a further year.

The focus of MARM is on building infrastructure and networking in mathematics in Africa, as well as supporting collaboration between mathematicians in non-African countries with a strong mathematical infrastructure and their African colleagues and students. Its wider aim is to foster long-term collaborative relationships between colleagues in UK/Europe and Africa.

Since 2005, 25 mentoring relationships have previously been supported through a MARM project, in Cameroon, Congo, Ethiopia, Ghana, Côte d'Ivoire, Kenya, Morocco, Namibia, Nigeria, Rwanda, Senegal, South Africa, Tanzania and Uganda. This further initiative aims to build on these successes and to continue to support mathematics in Africa.

Applications are now invited for the next round of MARM partnerships:

- From mathematicians based in UK/European universities:
  lms.ac.uk/grants/marm/marm-call-europe-2024

- From mathematicians based in African universities: lms.ac.uk/grants/marm/marm-call-africa-2024.

See further details on the scheme at lms.ac.uk/grants/marm. Read frequently asked questions at lms.ac.uk/grants/MARM/FAQs.

LMS Undergraduate Summer School 2025: Call for Expressions of Interest

The deadline for the call for expressions of interest to host the LMS Undergraduate Summer School in 2025 has been extended to 15 May 2024.

A grant of up £25,000 plus income from registration fees (£250 per registered student attending in-person and £25 per registered student attending remotely) is available to support the costs of an LMS Undergraduate Summer School that can accommodate at least 50 undergraduate students attending in-person and up to 200 undergraduates attending remotely.

The LMS has held an annual Undergraduate Summer School, aimed at introducing enthusiastic undergraduate students to modern mathematical research, since 2015. The Summer Schools take place for a two-week period in July and have proved very popular.

For more information and to submit an expression of interest, please visit: lms.ac.uk/events/lms-summer-schools.

VISITS

Visit of Swarnendu Banerjee

Dr Swarnendu Banerjee will be visiting the Universities of Sheffield, Leeds and York from 3 June to 14 June 2024. Dr Banerjee is a researcher at the Dutch Institute for Emergent Phenomena and member of the Theoretical and Computational Ecology group at the Institute for Biodiversity and Ecosystem Dynamics, University of Amsterdam. Dr Banerjee's recent research involves the analysis of tipping points and pattern formation in spatial ecosystems, as well as how ecosystems can mitigate the dangers of tipping points. Dr Banerjee will be giving lectures at:

- University of Sheffield, 3 June (contact Alex Best: a.best@sheffield.ac.uk)
- University of Leeds, 6 June (contact Francesca Scarabel: F.Scarabel@leeds.ac.uk)
- University of York, 12 June and 13 June (contact Brennen Fagan: brennen.fagan@york.ac.uk)

For further details contact Brennen Fagan (brennen.fagan@york.ac.uk). The visit is supported by an LMS Scheme 2 grant.

Visit of Florent Baudier

Professor Florent Baudier will be visiting the University of Cambridge from 9 to 21 June 2024. Professor Baudier is a member of the Functional Analysis group at Texas A&M University in College Station. His research areas include Metric and Banach Space Geometry with applications to Topology and Theoretical Computer Science. During his visit, Professor Baudier will give lectures at:

- University of Oxford, 11 June (contact Cornelia Drutu: cornelia.drutu@maths.ox.ac.uk);
- University of Cambridge, 19 June (contact András Zsák: a.zsak@dpmms.cam.ac.uk);
- University of Warwick, 20 June (contact Keith Ball: K.M.Ball@warwick.ac.uk)

For further details contact András Zsák (a.zsak@dpmms.cam.ac.uk). The visit is supported by an LMS Scheme 2 grant.

The Impact of LMS Grants on Education

The LMS Education Committee is dedicated to mathematics education, its most popular grant scheme being Small Grants for Education, which offers a one-off grant of up to £800 to fund an activity that stimulates interest and enables involvement in mathematics at any level from Key Stage 1 (i.e. age 5+) to Undergraduate level where there is no recourse for alternative sources of funding.

In the past three years, the Committee has given grants to over 40 different mathematics education activities benefitting thousands of children across the United Kingdom. These include Ravenshall School in Dewsbury, West Yorkshire, where over £500 was granted for resources to help pupils with sensory needs. “[They] have enjoyed using the tactile numbers in their learning and being able to take maths outside to learn has made a difference to all pupils using the resources with them being able to be more relaxed and calm[er] in their surroundings,” said Christine Gray-Sharpe, the Fundraising Manager at Ravenshall School.

At Pye Bank School in Sheffield, funding was given towards a school trip for thirty Year 5 and Year 6 pupils to visit the Maths City learning centre in Leeds. The students, where 54% of the pupils come from a ‘disadvantaged’ background according to the grant lead, got to take part in games and challenges that helped to develop their problem-solving skills which are part of the mathematics curriculum, as well as allowing them to experience mathematics outside of the classroom for the first time.

The grants have also enabled Belmont Junior School in Haringey, the fourth most deprived borough in London, to celebrate World Maths Day in March 2023, by funding the visit of the Problem Solving Company, a leading provider of immersive mathematics workshops and activity days. After the success of the previous year, the company returned with a tailored and individualised plan so that the children (pictured left and below) would get new experiences on the day, bringing in a varied range of puzzles. Mathematical concepts that they covered were understanding 2D and 3D shapes, logical thinking, spatial awareness, and code-breaking, to name a few. “It was a huge success and something that we are now [looking] to repeat,” said Diksha Shah, Acting Assistant Head (Curriculum) at the school.

For Holbrook Primary School in Trowbridge, Wiltshire, a £400 grant contributed to an entire day of Maths fun for all 245 pupils at the school. Maths workshops were delivered by Marvellous Magical Maths and pupils were immersed in mathematical activities all day across the school, with each pupil receiving a special commemorative badge at the end of the day’s festivities. “We are so grateful to all at the London Mathematical Society for making this opportunity possible for Holbrook children,” said Rosie Nunn, Deputy Headteacher at Holbrook School.

Kieran O’Connor
LMS Events Coordinator

LMS Council Diary — A Personal View

Council met via videoconference on Friday 2 February 2024. After the President had welcomed everyone, and particularly the new members to their first Council meeting, he noted with sadness the recent deaths of Dr Tony Gardiner and Professor Nicholas Higham FRS, both of whom had made significant contributions to the Society as well as the wider mathematics community. The President's business also included updates on the Protect Pure Maths Campaign, with a Maths Summit in the Science Museum and a parliamentary round table on mathematics provision in universities planned in the near future, and on the formation of an LMS Global Engagement Working Group in line with the LMS Strategy for 2023-2028.

Council then discussed the LMS-ICMU (International Centre for Mathematics in Ukraine) Distinguished Fellowship Scheme, after which there was an update from the LMS Academy for the Mathematical Sciences Working Group. It was reported that the Government consultation on a National Academy for the Mathematical Sciences had been launched with a deadline later in February, to which CMS (The Council for the Mathematical Sciences) would co-ordinate a response from the various learned societies, and that the President and Vice-President Gordon would attend a round table event as part of the consultation.

After the Executive Secretary highlighted that the LMS Strategy for 2023-2028 had been published on the LMS website, and that part of business planning would involve deciding how to support this strategy through the agreed Strategic Development Funds, there was a very interesting discussion on the importance of diversity in LMS prizes. It was agreed that there should be a campaign to target increasing the number and diversity of nominations.

Other business included the Treasurer's report on the first quarter review, a presentation of the Scrutineers Report by Charles Goldie on the recent Council elections, and presentation of various Committee Annual Reports. The Education Secretary highlighted the recently published Government consultation on an Advanced British Standard (ABS), a new qualification framework for 16-19 year olds, to which the Society would respond, while the chair of Research Grants Committee reported that following a drop in applications during Covid, applications had now increased to above pre-pandemic levels, which stimulated a discussion on the success rates of grants awarded by the Research Grants Committee and the Early Career Research Committee, both of which are clearly valued by the community.

The meeting concluded with the President thanking everyone for their contributions.

Elaine Crooks
LMS Council Member-at-Large

Maximising Your Membership: The Verblunsky Members' Room

All LMS members are invited to visit De Morgan House in Russell Square, London, and make use of the LMS Members' Room, or Verblunsky Room. Members are welcome to use the room during office hours, Monday to Friday 9.30–5.00pm. Please contact us by email on membership@lms.ac.uk to arrange a visit.

The Verblunsky Members' Room also houses two special collections: the Hardy Collection and the Philippa Fawcett Collection. The Hardy Collection, named after the Society's former President, contains over 300 volumes from G.H. Hardy's personal library of books. The Philippa Fawcett Collection, named after the first woman to come top in the Cambridge Mathematical Tripos exams and an early member of the LMS, is a wide-ranging library of books written by and about women who studied or worked in mathematical subjects in the nineteenth and first part of the twentieth century, or earlier.

Visiting members can also make use of the nearby LMS library housed at University College London. For details on how to register/renew as a UCL library user, see lms.ac.uk/library/how-register.

Valeriya Kolesnykova
LMS Accounts, Membership & Fellowship Assistant

Mathematics of Climate Models

EMILY POTTER

Scientists can use weather stations, satellite images, and records from ice, sediment and fossils to understand past climate. Future climate, however, requires some form of computer model to predict it. In this article I briefly discuss the mathematics behind climate models and their uses.

Introduction

There are many tools to understand weather and climate. In the recent past we can use observations such as those from weather stations, weather balloons, radar and satellites. To understand the climate from hundreds to millions of years ago, scientists measure the thickness of tree rings, the composition of air bubbles trapped in ice deep below the ground, and the material found in sediment cores. To predict future climate, however, scientists must use some form of mathematical model.

These climate models are also used to fill in gaps in our understanding of past climate, where observational data are not available. In addition, they can be used for experiments to determine what the earth's climate would look like under different circumstances, for example, without the addition of greenhouse gases produced by humans. The climate models which allow us to determine the average temperatures in one hundred years time are fundamentally similar to the models used by weather centres around the world to tell us the likelihood of rain tomorrow.

Climate models

At their core, dynamical climate models are based on the equations for the conservation of mass and momentum, a set of partial differential equations which govern the relationship between the velocity, pressure and density of a moving fluid such as air or water. An initial condition of the state of the climate is given, along with boundary conditions representing the oceans and land shapes (including mountains). This set of differential equations cannot be solved explicitly. Instead, the globe is split into a three-dimensional grid, and the equations are solved using numerical integration schemes.

This discrete grid, and the numerical integration schemes which are solved on it, present a limitation that all climate modellers have to contend with. In order for the numerical scheme to converge to a finite solution, the size of the timestep used for integration must be limited by the size of the three-dimensional grid, to fulfill the Courant-Freidrichs-Lewy (CFL) condition. The CFL condition is necessary for stability of a numerical solution to a differential equation, and it ensures that the timestep is small enough to capture the fastest waves in the solution to the differential equation.

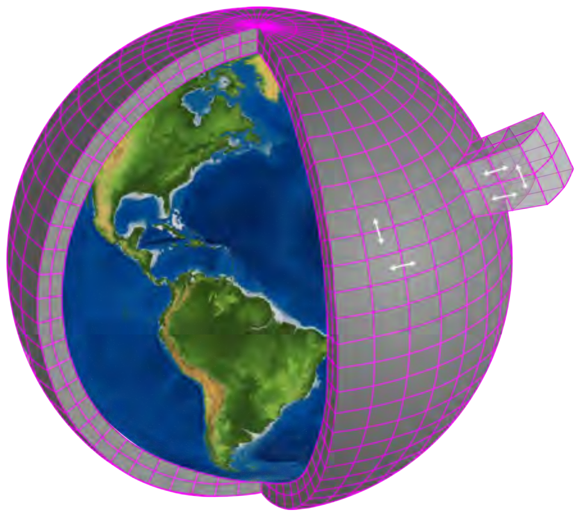


Figure 1. Global climate models split the world into a 3D grid, and the equations and physics are calculated at each grid point. Information is exchanged between rows and columns at each timestep. Image inspired by NOAA

In practice, this means that in order to run a model with a smaller grid, a smaller timestep is required. Thus it requires more computational power to predict one day into the future if you use a horizontal 5 km by 5 km grid than if you use a 50 km by 50 km grid, even ignoring the fact that you need more points to cover the earth using a 5 km grid. This is known as 'horizontal resolution' in climate models.

The state-of-the art global climate models currently run at a horizontal resolution of approximately 100 km. In the atmosphere, this captures large-scale weather features such as high- and low-pressure systems, monsoon winds and jet streams.

Aspects of climate which are not represented by the model dynamics (the modified mass and momentum conservation equations), are normally known as 'model physics'. These model physics cover a range of processes, including the impact of various greenhouse gas emissions and other atmospheric chemistry, changes of state of moisture from water vapour to cloud droplets and rain, and fluxes of moisture and heat between the ground and the atmosphere. The model physics tend to be statistical in nature, and based on an empirical understanding of each process. The complexity of the model physics can vary greatly between different climate models, and scientists may pick different physics options within a climate model depending on what they wish to achieve.

Many climate research centres around the world (often connected to national meteorological offices) produce historical and future climate simulations covering a few hundred years. They include both the atmospheric climate models described above, and ocean models. There is a large international collaboration which brings together these global simulations of the earth's climate, known as the Coupled Model Intercomparison Project (CMIP). The so-called CMIP models get updated every few years with new scientific and computing advances. The International Panel for Climate Change (IPCC) use averages from these models to make predictions about future climate change under various possible future human behaviour scenarios related to greenhouse gas emissions.

Historical notes on weather and climate modelling

Lewis Fry Richardson

Today, large climate models and weather forecasts are run on some of the most powerful supercomputers in the world. But incredibly, the first weather forecast was performed years before the first modern computers. Lewis Fry Richardson (1881–1953) was the first person to attempt to 'forecast' the weather based on dynamical equations. In 1916, he started to compute a 'retrospective' weather forecast for a day back in 1910 using a pen and paper. It took weeks to compute a 6 hour forecast, and unfortunately produced wildly unrealistic results. A strong believer in integrity, Richardson published these results regardless. Later analysis has found that the error was simply a result of not smoothing small discontinuities in the initial conditions. Fundamentally, Richardson's methods were correct and are essentially similar those used today.

Numerical schemes for solving partial differential equations

To give a very simple example of a numerical scheme, suppose we wanted to apply a numerical scheme to the one-way wave equation (1), where u represents velocity, x is the spatial coordinate, t the time coordinate, and $c > 0$ is the wave speed.

$$\frac{\partial u(x,t)}{\partial t} - c\frac{\partial u(x,t)}{\partial x} = 0 \qquad (1)$$

with initial condition $u(0,x) = u_0(x)$.

We could write a finite different scheme to approximate this equation at time n, and spatial point i, such as equation (2), where Δt is the timestep and Δx the grid spacing.

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} - c\frac{u_{i+1}^n - u_i^n}{\Delta x} = 0. \qquad (2)$$

In this case, the Courant–Friedrichs–Lewy (CFL) condition is given by $c\frac{\Delta t}{\Delta x} \leq 1$. This is a necessary condition for stability.

Regional climate modelling in mountains

Global climate models can provide broad averages about global climate, but there are many processes which are not captured at horizontal resolutions of 100 km. For example, clouds, especially convective clouds typical in tropical regions of the world, may be a few kilometers to tens of kilometers across. Atmospheric turbulence is on an even smaller scale, of tens to hundreds of meters. In global climate models, these smaller-scale processes would be parameterised in the model within the model physics, to keep the average conditions approximately correct.

Mountainous regions are particularly difficult to represent in global climate models, because the mountains cause very abrupt changes in weather and climate (it might be raining on one side of the mountain and not the other, for example). At the resolution of global climate models, where there is only a grid point every 100 km or so, many mountain ranges are very poorly represented, and as such the lower boundary condition to the model is substantially different from the real world.

Figure 2. A valley in Peru with clouds changing over a small spatial distance

To produce more accurate climate models for a specific region, climate modellers will embed a higher-resolution climate model within the global climate models. This is known as 'regional climate modelling', and these regional climate models are generally run at a resolution of a few kilometers. This allows some of the parameterised processes to be calculated by the dynamics of the model, and for a much more accurate representation of the land surface. In principal, this leads to more accurate climate predictions over that region.

My research uses regional climate models to understand the atmospheric processes occurring over mountain regions, their interaction with the land, and how the climate might change in the future. Using a regional climate model to create a dataset of temperature and precipitation over some regions of the Andes in Peru, I have determined the changes which have occurred over the past 40 years. By combining these regional climate models with statistical methods and data from the CMIP models, we were able to predict the warming that may occur over this century, and the likely changes in precipitation. This climate data is then used in collaboration with colleagues to predict future changes to glaciers and ultimately water resources.

Regional climate models can also be used for experiments. For example, I wanted to know how glaciers melting in a Himalayan valley might affect the temperature, wind and snowfall near the surface. Rather than wait decades for the glacier to melt, we can run the regional climate model with and without the representation of the glacier at the land boundary to determine these changes.

The weather is a chaotic system, and this can make it intrinsically very difficult to model. Despite this, climate models now form a fundamentally important part of weather and climate prediction and understanding. Climate change predictions also grapple with the difficulty of predicting future human behaviour, which can lead to large uncertainties in output. But without modelling projections, we would have no way to prepare for the changing climate over the next decades. As the statistician George Box said, "All models are wrong, but some are useful".

Acknowledgements

This work is funded by the Deplete and Retreat grant with the United Kingdom Research Institute, grant number NE/X004031/1.

Emily Potter

Emily is a postdoctoral researcher at the University of Sheffield. Her professional interests cover all aspects of mountain climate, particularly over the Himalaya and Andes mountain ranges. She also has a personal interest in mountains, and in her spare time can be found hiking up the nearest hill.

FURTHER READING

[1] Peter Lynch. The emergence of numerical weather prediction: Richardson's dream. Cambridge University Press, 2006.
[2] Lloyd Nicholas Trefethen. Finite difference and spectral methods for ordinary and partial differential equations. 1996.

Noetherian Extensions of Commutative Rings

HALEH HAMDI

In this article, we focus on an extension of commutative rings $R \subseteq T$ in which each ideal I of R with $IT = T$ is finitely generated. Also, we examine Lasker's decomposition theorem and Hilbert's basis theorem in the context of commutative ring extensions.

The class of Noetherian rings is extremely important in the contexts of commutative algebra and algebraic geometry. A commutative ring R is *Noetherian* if the property of the ascending chain condition for ideals of R is satisfied. However, now consider $S = R[X_1, X_2, \ldots]$, which is a polynomial ring in infinitely many variables over a commutative ring R. It is obvious that S is *not* Noetherian because the ascending chain of ideals $(X_1) \subsetneq (X_1, X_2) \subsetneq (X_1, X_2, X_3) \subsetneq \ldots$ does not stabilize.

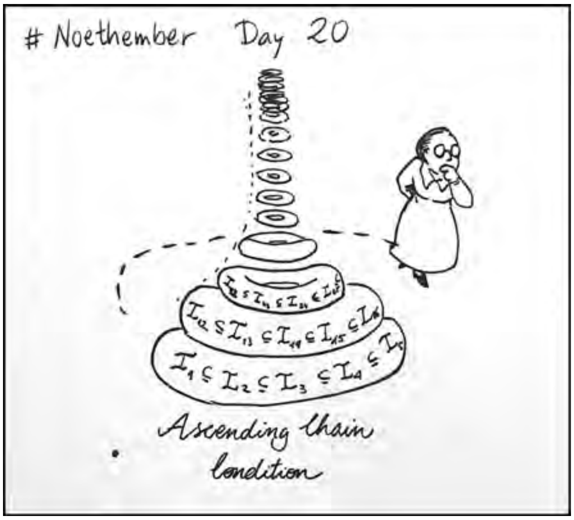


Figure 1. Ascending chain condition, source: www.crojasmolina.com

But the fascinating aspect of S is that it satisfies some well-known properties of Noetherian rings such as the principal ideal theorem, the primary decomposition property, the Hilbert basis theorem, the Mori–Nagata theorem, and the Krull intersection theorem for a certain class of ideals. In fact, S is a *strong Mori domain*, an integral domain in which the ascending chain condition holds on w-ideals [13].

Prime, primary, and minimal prime ideals

Let R be a commutative ring with identity. A proper ideal Q of R is called *prime* (respectively, *primary*) if $xy \in Q$ implies that $x \in Q$ or $y \in Q$ (respectively, $x \in Q$ or $y^n \in Q$ for some positive integer n).
A prime ideal P of R is said to be a *minimal prime ideal* over an ideal I of R if it is minimal among all prime ideals containing I.

The example of this ring S shows us that generalizations of Noetherian rings are highly significant. Furthermore, let us draw attention to one of the well-known properties of Noetherian rings, the principal ideal theorem (PIT) which states that over a Noetherian ring R, if x is a nonzero non-unit element of R, and P is a prime ideal of R minimal over (x), then the maximum length of chain of prime ideals of R is one, which we write as $\mathrm{ht}(P) = 1$. There are numerous examples of non-Noetherian rings in which the PIT fails. Strong Mori domains, as the most important generalization of Noetherian rings, are among the few classes of non-Noetherian rings that satisfy the PIT.

Analyzing analogues of some other properties of Noetherian rings over their generalizations is the inspiration for the work of many researchers over the years. In this article, we review one recent generalization of Noetherian rings and explore some of its basic properties.

Historical Background

One of the most significant milestones in modern algebra was the advent of rings in which any strictly ascending chain of ideals terminates, equivalently, rings in which each ideal is finitely generated.

Thanks to the appearance of these rings in the paper "Idealtheorie in Ringbereichen" [11], they were named *Noetherian rings*, honouring Emmy Noether, one of the most important female scientists of the 20th century. Emmy Noether was inspired by earlier works of Emanuel Lasker (1868–1941) who studied rings in which every ideal can be decomposed as an intersection of finitely many primary ideals. Lasker's theorem concerns polynomial rings over the field of complex numbers. In [11], Noether extended Lasker's decomposition theorem (for ideals in a ring of polynomials) to arbitrary Noetherian rings.

Figure 2. Emmy Noether, 1882-1935, source: [12, page 19]

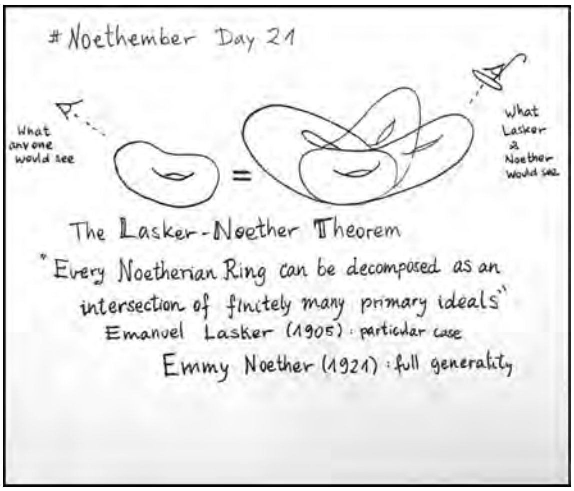


Figure 3. The Lasker-Noether Theorem, source: www.crojasmolina.com

This major breakthrough initiated the modern development of commutative algebra. She built her proof on a new elucidation of Hilbert's finite basis theorem, which had been published by David Hilbert (1862-1943) thirty years earlier in the context of invariant theory. In Noether's language this theorem states that ideals in every polynomial ring over a Noetherian ring are finitely generated.

Early in 1950, Irvin Cohen (1917-1955) showed that it is enough to focus on prime ideals of a commutative ring R to see whether R is Noetherian. He proved that if all prime ideals of R are finitely generated, then R is a Noetherian ring [4].

The other important feature of Noetherian rings has been proven by Irving Kaplansky (1917-2006). In 1974, he showed that every ideal of a Noetherian ring has only finitely many minimal prime ideals.

Noetherian Extensions

An extension T of a commutative ring with identity $R \subseteq T$ is called a *Noetherian extension* if every ideal I of R with $IT = T$ is finitely generated. We recall that IT is the ideal generated by I in T, and hence $IT = T$ means that each element t of T can be written as $\sum_{i=1}^{n} a_i t_i$ for some $a_i \in I$ and $t_i \in T$.

As a simple example of a Noetherian extension, one can take the extension $R \subseteq R[X]$ in which R is an arbitrary commutative ring, X is an indeterminate over R, and $R[X]$ is the polynomial ring with coefficients in R. We now construct an example of a Noetherian extension $R \subseteq T$ in which R is not a Noetherian ring by means of a pullback diagram [1].

Let $\mathbb{Z}$ be the ring of integers, $\mathbb{Q}$ the field of rational numbers, X an indeterminate over $\mathbb{Q}$, $T := \mathbb{Q}[X]$ the polynomial ring over $\mathbb{Q}$, $\phi : \mathbb{Q}[X] \to \mathbb{Q}$ the canonical homomorphism, and $R := \phi^{-1}(\mathbb{Z})$ the pullback of the following diagram:

$$\begin{tikzcd} R \arrow[r] \arrow[d] & \mathbb{Z} \arrow[d] \\ \mathbb{Q}[X] \arrow[r, "\phi"] & \mathbb{Q}. \end{tikzcd}$$

Indeed, $R = \mathbb{Z} + X\mathbb{Q}[X] = \{f \in \mathbb{Q}[X] \mid f(0) \in \mathbb{Z}\}$. Then $R \subseteq T$ is a Noetherian extension. To see this, we first note that $T = R_S$, the localization of R at S where S is the set of nonzero integers. Let I be an ideal of R with $IT = T$. Then $I \cap S \neq \emptyset$. Note that an ideal I of R intersecting S is of the form $A + X\mathbb{Q}[X]$ for some nonzero ideal A of $\mathbb{Z}$. In fact, $A = I \cap \mathbb{Z}$, and so $I = I \cap \mathbb{Z} + X\mathbb{Q}[X] = (I \cap \mathbb{Z})R$. Since $\mathbb{Z}$ is a principal ideal domain, A is a principal ideal, and hence I is finitely generated. This implies that $R \subseteq T$ is a Noetherian extension.

Localization

Let R be a commutative ring with identity and S a *multiplicatively closed subset* of R, that is, $1_R \in S$ and for $a, b \in S$, $ab \in S$. The *localization of R at S*, denoted $S^{-1}R$ or R_S, is the set of equivalence classes of pairs (r, s) with $r \in R$ and $s \in S$ such that

$$(a, s) \sim (a', s') \Leftrightarrow \text{there is an element } t \in S \text{ with } t(as' - a's) = 0.$$

The equivalence class of (a, s) is denoted by $\frac{a}{s}$. We make $S^{-1}R$ a ring by defining the following addition and multiplication:

$$\frac{a}{s} + \frac{b}{t} = \frac{at + bs}{ts} \quad \text{and} \quad (\frac{a}{s}).(\frac{b}{t}) = \frac{ab}{st},$$

for $a, b \in R$ and $s, t \in S$.
An ideal J of R_S is of the form $J = (J \cap R)R_S$, and if I is an ideal of R, then

$$IR_S = \{\frac{a}{s} \mid a \in I, s \in S\}.$$

Moreover, $IR_S = R_S$ if and only if $I \cap S \neq \emptyset$.
A simple example is the construction of the ring of rational numbers $\mathbb{Q}$ from the ring of integers $\mathbb{Z}$. We note that $\mathbb{Q} = \mathbb{Z}_S$ where S is the set of nonzero integers.
A ring with just one maximal ideal is called a *local ring*. Observe that the ring constructed by means of localization is not always a local ring. For instance, let $R = K[X]$ be a polynomial ring over a field K and $S = \{X^i \mid i \in \mathbb{N} \cup \{0\}\}$, where $\mathbb{N}$ is the set of natural numbers. Then R_S is not local since $M_1 = (X - 1)R_S$ and $M_2 = (X - 2)R_s$ are two distinct maximal ideals of R_S.
For a nonzero prime ideal P of R, the set $R \setminus P$ is a multiplicatively closed subset of R and we always denote $R_{R \setminus P}$ by R_P, which is a local ring with the maximal ideal PR_P.

On the other hand, R cannot be a Noetherian ring [1]. Indeed, if R is a Noetherian ring, then $M = (X)$ the ideal generated by X in T is finitely generated as an R-module and T-module, and hence $M^2 \neq M$. Otherwise, if $M^2 = M$, then by localizing both sides at $T \setminus M$, we get $M^2 T_M = MT_M$. Hence, $MT_M = 0$ by Nakayama's lemma. It implies that $M = 0$ since $M \subseteq MT_M$; a contradiction. Therefore, M/M^2 is nonzero $\mathbb{Q}$-module which is finitely generated $\mathbb{Z}$-module. Hence, M/M^2 is a vector space. Since every vector space is a free module, M/M^2 can be written as direct sum of copies of $\mathbb{Q}$. This implies that $\mathbb{Q}$ is a finitely generated $\mathbb{Z}$-module which cannot be true because $\mathbb{Q}$ as $\mathbb{Z}$-module is not Noetherian.

Nakayama's lemma

Assume that M is a finitely generated module over a commutative ring R and $J(R)$ is the intersection of all maximal ideals of R. If $MJ(R) = M$, then $M = 0$.

Ascending Chains in Noetherian Extensions

We know that if each nonzero ideal of a commutative ring R is finitely generated, then the ascending chain condition holds. Now, we examine the similar property on Noetherian extensions of commutative rings. Assume that $R \subseteq T$ is a Noetherian extension, and $I_1 \subseteq I_2 \subseteq \ldots$ is an ascending chain of ideals of R with $I_i T = T$ for each $i \geq 1$. We claim that this chain is stationary. To see this, let $K = \bigcup_{i=1}^{\infty} I_i$. It is clear that K is an ideal of R with $KT = T$. Hence, K is finitely generated. Let $K = (x_1, \ldots, x_t)$. For each $i = 1, \ldots, t$, there exists positive integer n_i such that $x_i \in I_{n_i}$. Take n as the maximum of all $n_1, \ldots, n_t$. Then for each $i = 1, \ldots, t$, $x_i \in I_n$. This implies that $I_i = I_n$ for each $i \geq n$.

Cohen's Theorem for Noetherian Extensions

Cohen's theorem is also valid for a Noetherian extension $R \subseteq T$. If every prime ideal P of R with $PT = T$ is finitely generated, then $R \subseteq T$ is a Noetherian extension. To see this, let Λ be the set of ideals of R with $IT = T$ which are not finitely generated, and assume that $\Lambda \neq \emptyset$. It is clear that if $I \subseteq J$ are ideals of R with $IT = T$, then $JT = T$. Hence, by Zorn's lemma, Λ contains a maximal element, say I. Then I is not a prime ideal, so that there are some $a, b \in R \setminus I$ with $ab \in I$. Now, I is a strict subset of $I + bR$, and $I + bR$ is finitely generated. Hence, we can choose $x_1, \ldots, x_n \in I$ such that $I + bR = (x_1, \ldots, x_n, b)$ and $(x_1, \ldots, x_n)T = T$. Note that

$$(I :_R bR) := \{x \in R \mid xbR \subseteq I\}.$$

Thus, $I + aR \subseteq (I :_R bR)$, and hence $(I :_R bR)$ is also finitely generated and $(I :_R bR)T = T$. Let

$$(I :_R bR) = (y_1, \ldots, y_m)$$

for some $y_i \in R$. Then

$$I = (x_1, \ldots, x_n, by_1, \ldots, by_m).$$

Hence, $I \notin \Lambda$, a contradiction. Therefore, $\Lambda = \emptyset$.

Zorn's lemma

Let S be a partially ordered set. If every totally ordered subset of S has an upper bound, then S contains a maximal element.

Kaplansky's Theorem for Noetherian Extensions

We now study the Noetherian extension analog of Kaplansky's theorem. If $R \subseteq T$ is a Noetherian extension, then every ideal I of R with $IT = T$ has only finitely many minimal prime ideals. To see this, let I be an ideal of R with $IT = T$. Then R/I is a Noetherian ring, and hence R/I has only finitely many minimal prime ideals. Thus, I has only finitely many minimal prime ideals.

Primary Decompositions of Proper Ideals in Noetherian Extensions

The most significant feature of Noetherian rings states that every proper ideal has a primary decomposition, that is, if I is a proper ideal of a Noetherian ring R, then $I = Q_1 \cap \ldots \cap Q_n$ for some primary ideals $Q_1, \ldots, Q_n$ of R. Now, we explore the analogue of this fact in the context of extensions of commutative rings. Assume that $R \subseteq T$ is a Noetherian extension. We claim that every ideal I of R with $IT = T$ has a primary decomposition. We first show that every proper ideal I of R with $IT = T$ is the finite intersection of irreducible ideals.

Irreducible ideals

A proper ideal I of a commutative ring R is *irreducible* if for any ideals I_1 and I_2 of R such that $I = I_1 \cap I_2$ either $I_1 = I$ or $I_2 = I$.

Assume to the contrary that this is not the case and let Λ be the set of all ideals of R such that their extension to T is T and they cannot be written as finite intersections of irreducible ideals. Since $R \subseteq T$ is a Noetherian extension and the set Λ is nonempty, Λ must contain a maximal element, say I. Hence, I is not irreducible, and hence there exist ideals I_1 and I_2 both properly containing I such that $I = I_1 \cap I_2$ and $I_i T = T$ for $i = 1, 2$. The maximality of I implies that I_1 and I_2 can be written as finite intersections of irreducible ideals in R. This implies that I can be also written as a finite intersection of irreducible ideals of R, contradicting that I belongs to Λ. Thus, every proper ideal I of R with $IT = T$ has a primary decomposition.

Now, we show that each irreducible ideal Q with $QT = T$ is primary. Take $a, b \in R$ such that $ab \in Q$, $b \notin Q$. We must show that $a^n \in Q$ for some positive integer n. For each positive integer i, put

$$I_i := (Q :_R a^i R),$$

in which $(Q :_R a^i R) = \{r \in R \mid ra^i \in Q\}$. Note that $I_1 \subseteq I_2 \subseteq \ldots$ is an ascending chain of ideals of R with $I_i T = T$ for each $i \geq 1$. Since $R \subseteq T$ is a Noetherian extension, there is a positive integer n such that $I_i = I_n$ for each $i \geq n$. Consider the ideals $I := (a^n) + Q$ and $J := (b) + Q$. We claim that $Q = I \cap J$. Clearly, $Q \subseteq I \cap J$. For the reverse containment, take $x \in I \cap J$. Then $x = ra^n + q$ for some $r \in R$ and $q \in Q$. Since $aJ \subseteq Q$, $ax \in Q$. Thus, $ra^{n+1} = ax - aq \in Q$. This implies that $r \in I_{n+1} = I_n$, and hence $x = ra^n + q \in Q$. Therefore, $I \cap J = Q$. Since Q is irreducible, $Q = I$ or $Q = J$. However, b not being in Q implies that $Q = I$, and hence $a^n \in Q$.

Hilbert's Basis Theorem for Noetherian Extensions

It is time to focus on Hilbert's basis theorem for extensions of commutative rings. Recall that if a commutative ring R is Noetherian, then every polynomial ring $R[X_1, \ldots, X_n]$ is also Noetherian. The Noetherian extension analog of this fact states that if $R \subseteq T$ is a weakly surjective Noetherian extension and X is an indeterminate over T, then $R[X] \subseteq T[X]$ is a Noetherian extension.

Weakly surjective extensions

As in [6], an extension $R \subseteq T$ of commutative rings is called *weakly surjective* if for every prime ideal P of R with $PT \neq T$, $R_P = T_{R \setminus P}$.

To prove Hilbert's basis theorem for extensions of commutative rings, it suffices to use Cohen's result. Let Q be a prime ideal of $R[X]$ with $QT[X] = T[X]$, and put $Q \cap R = P$. Assume that PT is a strict subset of T. Then $R_P = T_{R\setminus P}$ by assumption, and hence $T_{R\setminus P}[X] = QT_{R\setminus P}[X] = QR_P[X]$ which is a strict subset of $R_P[X] = T_{R\setminus P}[X]$, a contradiction. Thus, $PT = T$, and since $R \subseteq T$ is Noetherian, P (and hence $P[X]$) is finitely generated and R/P is a Noetherian ring. By the Hilbert basis theorem, $(R/P)[X] = R[X]/P[X]$ is Noetherian; so $Q/P[X]$ is finitely generated, and thus so is Q.

Mori-Nagata Theorem

It is natural to ask what other well-known results on Noetherian rings might appear in terms of commutative ring extensions. We would like to close this article with a short discussion of our research into the Mori-Nagata theorem on extensions of commutative rings [3]. The Mori-Nagata theorem is one of the most prominent theorems in commutative algebra. It states that the integral closure of a Noetherian domain is a Krull domain. An integral domain D is called a *Krull domain* if it satisfies the following three conditions:

(1) For every prime ideal P of D of height one, D_P is a discrete valuation ring.

(2) $D = \bigcap D_P$, where P ranges over all prime ideals of D of height one.

(3) Any nonzero element of D lies in only a finite number of prime ideals of height one.

The Krull dimension of a commutative ring

The *height* of a prime ideal P of a commutative ring R, denoted by $\mathrm{ht}(P)$, is defined to be the maximum of the lengths of chains;

$$P_0 \subsetneq P_1 \subsetneq \dots \subsetneq P_n = P$$

consisting of prime ideals of R.
The *Krull dimension* of R is defined as

$$\dim R := \sup\{\mathrm{ht}(P) \mid P \text{ is a prime ideal of } R\}.$$

In 1931, Krull domains were introduced by Wolfgang Krull (1899-1971) as a generalization of Dedekind domains: Noetherian integral domains which are integrally closed of dimension one. Over the years, the study of Krull domains has grown significantly.

The integral closure of a commutative ring

Let R be a commutative ring with total quotient ring K, that is, K is the localization of R at the set of non-zero divisors of R. The *integral closure* of R is the set of all element x in K such that x is a root of a monic polynomial over R. Then R is said to be *integrally closed* if R and its integral closure coincide.

Integral closures of Noetherian integral domains need not be Noetherian in general. However, the Krull-Akizuki theorem states that if the Krull dimension of an integral domain D is one, then every ring between D and its integral closure is Noetherian [10]. If the Krull dimension of D is two, then the integral closure is Noetherian, but there can be a non-Noetherian ring between D and its integral closure. If the Krull dimension of D is three, then the integral closure is certainly not Noetherian. In [7], Krull conjectured that in any case the integral closure of a Noetherian domain is a Krull domain. In [8], Mori affirmed the Krull's conjecture for local rings, and then this result was extended by Nagata to the general case in [9]. In the same paper, Nagata showed a detailed and slightly clearer proof of the result due to Mori.

The Krull-Akizuki theorem over extensions of commutative rings was studied in [2]. Let $R \subseteq T$ be a Noetherian extension, $x \in R$ with $xT = T$, and D a ring between R and $R^{g(T)}$ such that

$$R^{g(T)} = \{x \in T \mid xM_1 \cdots M_k \subseteq A \text{ for some } M_i \in \Delta\}$$

where Δ is a set of maximal ideals M of R with $MT = T$. Then D/xD is a Noetherian R-module and every ideal of D containing x is finitely generated. As a consequence, if $R \subseteq T$ is a weakly surjective Noetherian extension in which R possess at least one prime ideal P with $PT = T$, and each prime ideal of R whose extension to T equals to T is a maximal ideal, then $R^{g(T)} = T$. In particular, for each commutative ring D between R and T, $D \subseteq T$ is a Noetherian extension.

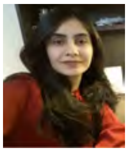
Acknowledgements

I would like to thank Professor Gyu Whan Chang, whose help was invaluable while I was visiting Incheon

National University during 2018 where the paper [2] was completed.

FURTHER READING

[1] Brewer, J.W., and Rutter, E.A. 1976. "$D + M$ constructions with general overrings". Michigan Mathematical Journal. 23(1), 33–42.
[2] Chang, G.W. and Hamdi, H. 2019. "Noetherian extensions of commutative rings". Journal of Algebra, 534, 344–357.
[3] Chang, G.W. and Hamdi, H. "Krull extensions of commutative rings". Preprint.
[4] Cohen, I.S. 1950. "Commutative rings with restricted minimum condition". Duke Mathematical Journal, 17(1), 27–42.
[5] Kaplansky, I. 1974. "Commutative rings", revised edition, University of Chicago Press, Chicago.
[6] Knebusch, M. and Zhang, D. 2002. "Manis valuations and Prüfer extensions I". Lecture Notes in Mathematics, 1791. Springer, Berlin.
[7] Krull, W. 1935. "Idealtheorie". Springer-Verlag, Berlin.
[8] Mori, Y. 1953. "On the integral closure of an integral domain", Memoirs of the College of Science, University of Kyoto. Series A: Mathematics, 27(3), 249–256.
[9] Nagata, M. 1955. "On the derived normal rings of Noetherian integral domains". Memoirs of the College of Science, University of Kyoto. Series A: Mathematics, 29(3), 293–303.
[10] Nagata, M. 1962. "Local Rings", Interscience, New York.
[11] Noether, E. 1921. "Idealtheorie in ringbereichen". Mathematische Annalen, 83(1–2), 24–66.
[12] Rowe, D.E. 2021. "Emmy Noether — Mathematician Extraordinaire". Springer Nature.
[13] Wang, F., and McCasland, R. L. 1999. "On strong Mori domains". Journal of Pure and Applied Algebra, 135(2), 155–165.



Haleh Hamdi

Haleh is a researcher with an interest in commutative rings and algebras. She received her Ph.D. from University of Tabriz, Iran. She is still advancing from novice to amateur in understanding the mathematics behind violins and life!

The Heidelberg Laureate Forum: Bringing Together Some of the Brightest Minds in Mathematics

MARIANNE FREIBERGER AND RACHEL THOMAS

How do you inspire the next generation of mathematicians? One way is to bring them together with the very best in the field — people whose work has been foundational, whose thoughts spark new ideas, and whose advice (and anecdotes) can be invaluable. Every year the Heidelberg Laureate Forum (HLF) offers just such an opportunity for generational exchange between young researchers from around the globe and laureates of the main prizes in mathematics and computer science. Whether you are a young researcher, someone seeking to encourage the early career researchers you work with, or even a laureate yourself, we would like to introduce you to the HLF and invite you to apply, encourage applications, and take part.

Sparking interactions

The HLF is organised by the Heidelberg Laureate Forum Foundation (HLFF) and takes place every year in September in the German university town of Heidelberg. Over the course of a week 200 young researchers, from undergraduate to postdoctoral level, mix and mingle with recipients of the Fields Medal and the Abel Prize in the field of mathematics, the ACM A.M. Turing Award and the ACM Prize in Computing in the field of computer science, and the IMU Abacus Medal and Nevanlinna Prize representing the overlap between the two fields.

Robert Endre Tarjan (1982 Nevanlinna Prize, 1986 ACM A.M. Turing Award) talking to young researchers at the 9th HLF 2022. Photo © Heidelberg Laureate Forum Foundation

The HLF offers the scientific lectures, workshops and poster sessions you would recognise from any scientific meeting. But in addition there are unique and welcoming opportunities for all participants to interact: panel discussions, speed networking sessions, discussions in small groups, and coffee breaks and meals allowing ample time for talk. There is also a busy social programme including a boat trip down the river Neckar (with opportunities for dancing), a Bavarian beer fest, and dinner at the Heidelberg castle. At all events the laureates mix with the young researchers, formally and informally, and in return enjoy a chance to see old friends and meet a diverse group of brilliant young mathematicians and computer scientists who will lead future research.

"One of the goals of the HLF is sparking scientific interactions between the laureates and the young researchers," says Anna Wienhard, Scientific Chair of the Heidelberg Laureate Forum Foundation. "And I say sparking because that scientific interaction may start in a lecture or a panel discussion, but then continue in the coffee break afterwards, or in a discussion over dinner. The other goal is to get mathematicians and computer scientists together to exchange and discuss the challenges within these fields and at their interface, and also to go beyond and see what is the role that mathematics and computer science plays in our society."

Young researchers from around the world

The HLF was initiated, and is funded, by the German foundation Klaus Tschira Stiftung, which promotes natural sciences, mathematics and computer science. The HLF was inspired by the Lindau Nobel Laureate Meetings, which support exchange between Nobel Laureates and young scientists.

The organisations which award the prizes that are being represented — the International Mathematical Union (IMU), the Norwegian Academy of Science and Letters (DNVA), and the Association for Computing Machinery (ACM) — are partners of the HLF. It's these organisations that choose the young researchers, who arrive from more than 50 countries each year. "The HLF provides the infrastructure for the applications by the young researchers, but the selection is in the hands of the award granting institutions," says Wienhard.

Applications from young researchers are invited between November and February each year. The 200 successful applicants are chosen in equal parts from mathematics and computer science, balancing the overlap between the fields, in a way that ensures geographical, gender and social diversity and equity. "It's not just young researchers from top universities that come to the HLF," says Wienhard. "The group is much more diverse than that, so laureates can interact with people they may not meet at ordinary scientific conferences."

To ensure a balanced choice, each application is read by three reviewers from a team chosen by the award granting institutions. The team is selected to ensure they have an understanding and experience of the geographical regions applicants come from, as well as the fields they are studying. And while quantitative measures, such as exam grades, are important, much weight is given to letters of recommendation and, crucially, the applicants' own letter of motivation. "The ability to ask good questions is vital," says Sergei Tabachnikov, Professor of Mathematics at Pennsylvania State University, who is involved in selecting applicants. "It's also important that they will be able to make the most of their opportunity to interact with other HLF participants when they are here."

The excitement among the young researchers each year is palpable. Laureate lectures and panel discussions themselves present rare opportunities, but being able to present lightning talks and posters to this eminent audience, and to have informal chats or even a little dance with a laureate, goes far beyond what most will have experienced in their academic life so far. And it creates a lasting impact. "There is a certain exposure to scientific ideas, topics, and discussions, which has an influence on the directions young researchers choose in the future," says Wienhard.

Equally important as interactions with laureates is the opportunity to connect with a select and interesting group of other young researchers. "One important goal is to create networks between young people, especially if they are geographically remote," says Tabachnikov. The HLF offers several projects to make sure the connections that are made within a week in September last far into the future, and that young researchers are supported beyond the event itself.

"Our network for young researchers to stay connected after the event is called AlumNode, a cooperation with the Klaus Tschira Stiftung and the German Scholars Organisation," says Sarah MacLeod, Head of Young Researcher Relations for the HLF. "It's an interdisciplinary alumni network the young researchers can join. They can apply for joint project funding and we offer them workshops, peer mentoring, and regular get togethers. We also have the HLFF Spotlight series where we highlight the happenings in the lives of our alumni." Additionally, in 2023 the HLFF Inspiring Minds project was launched, which provides expertise and guidance through an expert mentoring programme and a range of engaging digital formats designed to support alumni in developing their personal career path.

Bringing together the brightest minds

The content of the scientific programme is planned by the HLF scientific committee, which comprises international experts in mathematics and computer science and representatives of the award granting organisations and the Heidelberg Laureate Forum Foundation. All laureates are welcome to contribute to the programme. The exact nature of laureates' contributions is discussed in advance with the committee open to all sorts of propositions — from a traditional talk on a topic of a laureate's choice, to discussion groups, master classes, and visits to local schools.

A conversation between Ingrid Daubechies, Maryna Viazovska (2022 Fields Medal), Anna Wienhard and Karen Uhlenbeck (2019 Abel Prize) at the 10th HLF, 2023. Photo © Heidelberg Laureate Forum Foundation

Apart from sessions involving laureates, there are also activities focused on particular areas chosen by the committee. For example, each year a Hot Topic is examined through talks and discussions involving experts in the area: quantum computing, epidemic modelling, and deep learning are examples of fields that have featured as Hot Topics over recent years. Those participants who are not young researchers, and in particular the laureates, are attracted to the HLF by more than just the excellent food and accommodation, beautiful surroundings, and desire to support the next generation.

"The HLF is a chance to meet experts in mathematics and computer science, as well as very bright young people," says Efim Zelmanov who won a Fields Medal in 1994 and has attended almost every year. "It is also a fertile meeting ground of mathematics and computer science. The organisers' warm hospitality is another definite plus."

The interaction between mathematicians and computer scientists at all levels is indeed a key attraction of the HLF. "Very interesting things happen at the interface between maths and computer science and it's important to bring researchers together," says Wienhard.

Vinton Cerf, who received the ACM A. M. Turing Award in 2004 and has been involved with the HLF from the beginning, also enjoys the open invitation to all laureates of the prizes involved: "I got to see some old friends I hadn't seen in a long time and I keep getting to see them every year." In recent years new types of sessions, such as Laureate Discussions have allowed participants to share ideas and insights in different ways, aiming, for example, to identify fruitful interactions between mathematics and computer science, and examining topics such as the role of mathematical proof in computer science. The Laureate Discussions are designed with the laureates' input.

But it's not just the interaction with peers that laureates and other experts find invigorating. The young researchers who attend are not only among the brightest, they also come from regions of the world and social backgrounds that are not always represented at scientific conferences. "They bring problems to the table that are not easy to solve and even the laureates may not know the answers," Cerf said at the 9th HLF 2022. "I have problems with the systems I'm working on now and I love the opportunity to tell the students: here's the problem — do you have any good ideas for solving it? So this is very much a two way street."

Testimonies from the Young Researchers

"I enjoyed meeting the laureates a lot — asking questions and listening to their advice. I also had the possibility to network with young researchers and I think that many collaborations will arise from the conference." *Marithania Silvero, Young Researcher 8th HLF 2021, Spain*

"The HLF not only broadened my horizons by showcasing groundbreaking research but also provided a stimulating environment to exchange ideas, learn from laureates, and cultivate lifelong connections." *Oluwatosin Babasola, Young Researcher 9th HLF 2022, Nigeria*

"The HLF really transformed my thinking. Seeing all these challenges in maths and computer science gives me a lot of inspiration for the application of AI in health systems, especially in Africa." *Jimoh Abdulganiyu, Young Researcher 9th HLF 2022, Nigeria*

"After the HLF keep in contact with each other... Today I am still working with people I met in 2019, we're still collaborating on papers and research work." *Jie Li, Young Researcher 7th HLF 2019 and MC 9th HLF 2022, China* (Photo: Yuxuan Feng, Light Memory Studio)

Finally, the HLF provides an opportunity for the research community to celebrate those important prizes and so celebrate mathematics and computer science themselves.

Every year the HLF is accompanied by a public exhibition. This photo, taken at the 7th HLF in 2020, is from the exhibition 'La La Lab —The Mathematics of Music', which was developed by IMAGINARY. It shows Andreas Matt, Director of IMAGINARY, and musician and mathematician Charles Gray. Photo ©Heidelberg Laureate Forum Foundation

Inspiring the next generation

In September 2023 the HLF celebrated its 10th birthday. Recognising the growing impact of artificial intelligence on all our lives, the opportunity to bring together mathematicians and computer scientists was timely. Young researchers were particularly engaged with the sessions exploring AI and its role in society. This session sat alongside the eagerly anticipated lecture by Hugo Duminil-Copin whose work was recognised by a 2022 Fields Medal.

To mark the 10th HLF several new types of sessions were on offer. Lightning Talks were one-slide presentations by laureates looking back at the last ten years in research, some even risking a glance into the future. In Spark Sessions laureates explored current research in short talks, while master classes explored specialised topics such as randomness (with Abel laureate Avi Wigderson) and algebraic structures (with Efim Zelmanov). Twenty HLF alumni were also invited, with some moderating discussion rounds and even running a workshop on improvised theatre.

A total of 12 mathematics laureates attended the 10th HLF, including Fields Medallist Maryna Viazovska and Abel laureate Karen Uhlenbeck. With Viazovska attending in person and Uhlenbeck remotely, it was significant to have the only two living female laureates in mathematics speaking at the Forum. In an interview for the HLF Vlog Abel laureate László Lovász explained some of the benefits of the HLF for the laureates: "You can see that all over the world there are people who love your subject, who are devoted to science. So the future looks brighter than before meeting them."

Bavarian evening at the 9th HLF 2022. Photo © Heidelberg Laureate Forum Foundation

As the HLF enters its second decade, we would like to invite young mathematicians from around the world, as well as laureates, to apply and attend. As young researcher Narinder Sing Punn said at the 9th HLF 2022, "Apply for the HLF. It is a life changing experience — don't miss this opportunity."

All students and early career researchers in mathematics and computer science up to postdoctoral level can apply. The next application period for Young Researchers will begin November, 2024. To apply, please visit https://application.heidelberg-laureate-forum.org

This article first appeared in the Notices of the American Mathematical Society, DOI 10.1090/noti2834

Marianne Freiberger and Rachel Thomas

Marianne Freiberger and Rachel Thomas are the editors of plus.maths.org, part of the Millennium Mathematics Project at the University of Cambridge. Their email address is plus@maths.cam.ac.uk.

Notes of a Numerical Analyst

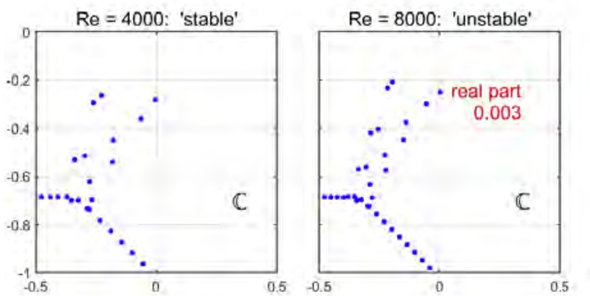
The Meaning of Eigenvalues

NICK TREFETHEN FRS

In mathematics, we idealize things. This is an essential and powerful part of what we do. But we must be on our guard to make sure the idealizations don't lose contact with the phenomena they are intended to shed light on.

Eigenvalues of nonsymmetric matrices and operators are a particularly extreme case of this challenge. In many applications, they don't have the significance we are trained to expect.

Let's look at possibly the most extensively studied nonsymmetric eigenvalue problem of all: the Orr-Sommerfeld equation. The topic is stability of fluid flows. The scientific question is, what makes high speed flows go turbulent?



To find answers, for more than a century, fluid mechanicians have investigated eigenvalues. The Navier–Stokes equations are nonlinear, and the eigenvalues are those of the linearization about a smooth, nonturbulent solution. The idea is that if all the eigenvalues are in the left half-plane, the flow is stable, whereas if there is an eigenvalue in the right half-plane, it is unstable and the unstable mode may grow into turbulence.

Consider the barely distinguishable plots above. The Reynolds number $\mathfrak{R}$ is the nondimensionalized speed, and on the left, at $\mathfrak{R} = 4000$, the flow is eigenvalue stable. On the right, at $\mathfrak{R} = 8000$, it is eigenvalue unstable. An incredible amount is known about these eigenvalues, and Steve Orszag got famous for calculating that the critical value at which one of them moves into the right half-plane is Re $= 5772.22$.

So the traditional view is that something suddenly changes when $\mathfrak{R}$ hits 5772.22. And of course, there is a theorem which proves that in a certain sense this is true.

Yet laboratory experiments almost never fit this picture. Actual flows don't show a sharp Reynolds number for transition to turbulence, and turbulence is often observed at both $\mathfrak{R} = 4000$ and 8000. The reason becomes clear if we consider the eigenvalue labeled in red. What exactly does it imply? It implies that flow perturbations can grow at rate $\exp(0.003t)$. By $t = 300$, such a perturbation will be amplified by a factor of e. This corresponds to a channel 300 times as long as it is wide, pretty much the limit of what can be built in the lab. And of course, amplification by e is not going to drive turbulence; you'll need more than that. So if you think about this image quantitatively, it is hardly surprising that the "instability" it represents is not observed.

The actual mechanism of transition to turbulence involves other parts of the spectrum, well in the stable left half-plane, associated with strong nonnormality, not shown in these pictures.

In fluid mechanics and in other applications, if an eigenvalue is in the right half-plane, this implies nothing about local behavior. The implications only concern the potential fate of certain trajectories if the system remains linear and unperturbed as $t \to \infty$. There are systems that behave like that, usually featuring matrices or operators that are symmetric or nearly so. But in plenty of other cases, certainly in high Reynolds number fluid mechanics, eigenvalue analysis has brought confusion. In areas like ecology and food webs, with all their complexities and time-dependencies, the idea of inferring anything precise from whether or not there are eigenvalues in the right half-plane is really very nebulous.

Nick Trefethen
Trefethen is Professor of Applied Mathematics in Residence at Harvard University.

Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

Shape morphing during evaporation

AUTHORS: Riccobelli, Al-Terke, Laaksonen, Metrangolo, Paananen, Ras, Ciarletta, Vella
ACCESS: https://arxiv.org/abs/2212.00826

How does the shape of a droplet of fluid change during evaporation? The answer depends on many factors. For example, droplets of a pure liquid maintain their shape whereas droplets containing solute generally do not. Surprisingly, this depends on gravitational effects—even when the droplets are very small. This paper, published in *Physical Review Letters* in 2023, considers droplets with solute in two different situations. First, the droplet is supported against gravity by a surface (sessile) and, second, the droplet is suspended beneath the surface (pendant). In the sessile case the droplet flattens parallel to the surface, and in the pendant case the droplet elongates perpendicular to the surface forming a vase-like shape. As well as being a theoretical breakthrough, this work has potential to inform applications in engineering and biomedical science where one needs to control the shape of small droplets.

Balls in groups: volume, structure and growth

AUTHORS: Romain Tessera and Matthew Tointon
ACCESS: https://arxiv.org/abs/2403.02485

Gromov's theorem is a seminal result in geometric group theory which states that a finitely generated group has polynomial growth if and only if it has a nilpotent subgroup of finite index (that is, it is 'virtually nilpotent'). Here 'polynomial growth' means that the number of elements of the group expressible as a product of n generators (from a fixed symmetric generating set) is bounded above by a polynomial function of n. Can our readers prove that the property of having polynomial growth is independent of the choice of generating set? Breuillard–Green–Tao proved a celebrated quantitative version of Gromov's theorem in 2012, where they quantify, for example, how the nilpotency class and index depend on the polynomial growth rate. This 94-page paper, appearing on arXiv in March 2024, proves a sharp version of Breuillard–Green–Tao's theorem. I had the pleasure of hearing one of the authors present this work in the pure maths colloquium in St Andrews right after the paper appeared on arXiv.

The largest prime factor of $n^2 + 1$

AUTHORS: Hector Pasten
ACCESS: https://arxiv.org/abs/2312.03566

A famous result in number theory due to Chowla (1934) is that the largest prime factor of $n^2 + 1$ is at least a fixed constant times $\log\log n$ for large enough n. This might seem like a modest lower bound but observe that replacing $n^2 + 1$ with e.g. 2^n we see that the largest prime factor of a reasonably defined increasing sequence of integers need not grow at all. Important here is that $n^2 + 1$ is a (non-linear) polynomial and in fact Chowla's result is now known to hold for far more general polynomials.

This paper, published in *Inventiones Mathematicae* in 2024, proves that the largest prime factor of $n^2 + 1$ is at least a fixed constant times

$$\frac{(\log\log n)^2}{\log\log\log n}$$

for large enough n. This significant improvement over the state of the art in Chowla's problem, as well as improvements on the state of the art in the related ABC conjecture, is achieved in under 13 pages!

The ABC conjecture is that for a given $\varepsilon > 0$ there is a constant $\kappa > 0$ depending only on ε such that if a, b and c are coprime positive integers with $a + b = c$, then c must be bounded above by $\kappa r^{1+\varepsilon}$ where r is the largest square-free divisor (the 'radical') of the product abc. My son is currently learning his ABCs but has yet to work seriously on this conjecture.

Jonathan Fraser is pictured here with collaborators underneath a large Cayley graph. Is the volume of an n-ball in the path metric bounded by a polynomial function of n?

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Intermediate Dimensions

AMLAN BANAJI

Hausdorff and box dimension are two notions of fractal dimension, and the intermediate dimensions are a family of dimensions which lie between them. In this microthesis, we describe the form that these dimensions can take for general sets and for a class of self-affine fractal sets.

Fractal geometry

Much of the classical study of geometry relates to smooth objects such as manifolds, which have a well-defined integer dimension. However, many natural phenomena, such as the British coastline, display much more detailed and intricate structure across a range of scales, and often have some form of *self-similarity*, meaning that small parts of it have similar properties to the whole. Objects with these properties are often called *fractals*, though the term has no precise mathematical definition. The 'length' of the British coastline increases as we decrease the length δ of the ruler used to measure it, and scales very roughly like a constant multiple of $\delta^{-0.2}$ (i.e. we need to use $\approx \delta^{-1.2}$ rulers of length δ) over a range of scales. It therefore makes sense to regard the 'dimension' of the coastline as being 1.2.

To describe the theory of fractal dimensions more formally, we work with non-empty bounded subsets of Euclidean space $\mathbb{R}^d$ throughout. The box dimension of such a set F is defined by

$$\dim_{\mathrm{B}} F := \lim_{\delta \to 0} \frac{\log N_\delta(F)}{-\log \delta}$$

(if the limit exists), where $N_\delta(F)$ is the smallest number of open balls of diameter δ needed to cover F. There is another notion of dimension, called *Hausdorff dimension*, which is a lower bound for box dimension, and is perhaps more widely used across mathematics. It is defined by

$$\dim_{\mathrm{H}} F := \inf\{s \geq 0 : \text{for all } \varepsilon > 0 \text{ there is a cover } \{U_i\}_{i=1}^{\infty} \text{ of } F \text{ such that } \sum_i (\mathrm{diam} U_i)^s \leq \varepsilon\}.$$

Intermediate dimensions

The key difference between the two dimensions discussed above is that all the sets in a cover for box dimension have equal diameter δ, whereas for Hausdorff dimension the diameters of covering sets lie in an interval $[0,\delta]$. The intermediate dimensions $\dim_\theta F$ are defined by restricting the allowable diameters of sets in the cover to intervals of the form $[\delta^{1/\theta},\delta]$, where $\theta \in (0,1]$ is a fixed parameter. They satisfy $\dim_{\mathrm{H}} F \leq \dim_\theta F \leq \dim_{\mathrm{B}} F$ with $\dim_1 F = \dim_{\mathrm{B}} F$, and we define $\dim_0 F = \dim_{\mathrm{H}} F$. These dimensions provide more nuanced geometric information than Hausdorff or box dimension in isolation.

Definition 1. (Falconer–Fraser–Kempton [3]). For a set $F \subset \mathbb{R}^d$, $\theta \in (0,1]$ and $s \in [0,d]$, define

$$S^s_{\delta,\theta}(F) := \inf\Big\{ \sum_i (\mathrm{diam} U_i)^s : \{U_i\}_i \text{ is a cover of } F \text{ such that } \delta^{1/\theta} \leq \mathrm{diam} U_i \leq \delta \text{ for all } i \Big\}.$$

If there exists s such that $\frac{\log S^s_{\delta,\theta}(F)}{-\log \delta} \to 0$ as $\delta \to 0$, then we say that s is the *intermediate dimension* of F at θ, and write $s = \dim_\theta F$.

If the limit in the definition of box or intermediate dimension does not exist then we can use limsup and liminf to define upper and lower versions of the dimensions. The function $\theta \mapsto \dim_\theta F$ is always non-decreasing in θ, and is continuous for $\theta \in (0,1]$ but not generally at $\theta = 0$.

For many simple sets the intermediate dimensions have a simple form with at most finitely many points of non-differentiability, for example $\dim_\theta(\{1/n : n \geq 1\}) = \theta/(1+\theta)$ (see [3]). In contrast, the following characterisation shows that for general sets, the intermediate dimensions can have highly varied behaviour. Recall that the upper Dini derivative of a function $f\colon \mathbb{R} \to \mathbb{R}$ at x is given by

$$\mathrm{D}^{+} f(x) := \limsup_{\varepsilon \to 0^{+}} \frac{f(x+\varepsilon) - f(x)}{\varepsilon}.$$

Theorem 1. *(Banaji–Rutar [2]). Let $h\colon [0,1] \to [0,d]$ be any function. Then there exists $F \subset \mathbb{R}^d$ with $\dim_\theta F = h(\theta)$ if and only if h is non-decreasing, is continuous on $(0,1]$, and satisfies*

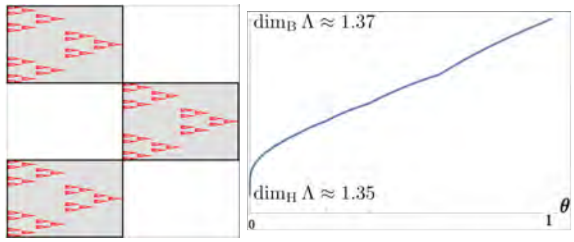
$$\mathrm{D}^{+} h(\theta) \le \frac{h(\theta)(d - h(\theta))}{d\theta} \qquad \text{for all } \theta \in (0,1).$$

It follows that if f is any non-decreasing Lipschitz function on $[0,1]$, there exist $F \subset \mathbb{R}$ and $a > 0$, $b \in \mathbb{R}$ such that $\dim_\theta F = af(\theta) + b$. In particular, the function can be non-differentiable at each point in a dense subset of $(0,1)$. A key step in the proof of Theorem 1 is the construction of a Cantor set with non-uniform subdivision ratios.

Bedford–McMullen carpets

An *iterated function system (IFS)* is a finite set of contraction maps $\{S_i\colon D \to D\}_{i=1}^{N}$, where $D \subset \mathbb{R}^d$ is closed. Given an IFS, there is a unique non-empty compact set Λ, called the *attractor*, satisfying $\Lambda = \bigcup_{i=1}^{N} S_i(\Lambda)$. Familiar fractals such as the middle-third Cantor set and the Sierpinski triangle are *self-similar sets*, which are the attractors of IFSs consisting of similarities (i.e. maps which contract distances by a constant ratio). The Hausdorff and box dimensions of self-similar sets always coincide, so we work with more general *self-affine sets*, where the contractions are affine. We work with the following particular class of sets, which have become a standard example in fractal geometry. Divide a square into an $m \times n$ grid, where $2 \le m < n$, and choose a subset of the rectangles. Write $\gamma := \log n / \log m$. Consider the IFS of maps which send the square onto each of the chosen rectangles, preserving orientation. The attractor Λ is called a *Bedford–McMullen carpet*, after the authors who independently introduced these sets in 1984 and calculated their Hausdorff and box dimensions (which typically differ).

In [1], the author and Kolossváry calculate a precise formula (unfortunately too complicated to state here) for the intermediate dimensions of all Bedford–McMullen carpets. The proof uses tools from probability, dynamics and information theory, and involves explicitly constructing a cover using scales $\delta, \delta^{\gamma}, \delta^{\gamma^2}, \dots, \delta^{\gamma^{L-1}}$ and $\delta^{1/\theta}, \delta^{1/(\gamma\theta)}, \dots, \delta^{1/(\gamma^{L-1}\theta)}$ when $\gamma^{-L} < \theta < \gamma^{-(L-1)}$ (for $L \ge 1$). The intermediate dimensions have a more complicated form than previously observed for other classes of fractals.



A Bedford–McMullen carpet and the graph of its intermediate dimensions. Note that the graph has countably many phase transitions (see [1]) and is continuous on $[0,1]$ (see [3]).

Theorem 2. *(Banaji–Kolossváry [1]). Let Λ be a Bedford–McMullen carpet with $\dim_{\mathrm{H}} \Lambda < \dim_{\mathrm{B}} \Lambda$. Then the function $\theta \mapsto \dim_\theta \Lambda$ is strictly increasing. Moreover, for all integers $L \ge 1$, this function is real analytic and strictly concave on the interval $(\gamma^{-L}, \gamma^{-(L-1)})$, but is non-differentiable at $\theta = \gamma^{-L}$.*

Our formula also has useful applications. In particular, we use it to give a necessary condition for there to exist a Lipschitz bijection with a Lipschitz inverse between two Bedford–McMullen carpets.

FURTHER READING

[1] A. Banaji, I. Kolossváry, Intermediate dimensions of Bedford–McMullen carpets with applications to Lipschitz equivalence, Preprint 2021, arXiv:2111.05625.
[2] A. Banaji, A. Rutar, Attainable forms of intermediate dimensions, Ann. Fenn. Math. 47 (2022) 939–960.
[3] K. J. Falconer, J. M. Fraser, T. Kempton, Intermediate dimensions, Math. Z. 296 (2020) 813–830.

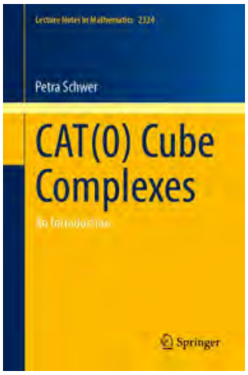
Amlan Banaji

Amlan grew up in London. He completed his PhD at the University of St Andrews in 2023, and is now a postdoctoral researcher at Loughborough University. His research interests relate to geometry, analysis and dynamical systems.

CAT(0) Cube Complexes: An Introduction

by Petra Schwer, Springer, 2024, £49.99, €58.84, ISBN: 978-3031436215

Review by Yuri Santos Rego



Triangles. Cubes. Curvature. Hyperplanes. Long ago, all geometers lived together in harmony, believing everything to be flat. Then, everything changed when non-Euclidean polygons attacked, such as Khayyam's hyperbolic quadrilaterals [4]. Only a solid geometric foundation, master of not just Euclid's Elements, could unite geometers, but when the mathematical world needed formalism the most, it (partially) vanished. Hundreds of years passed and our mathematical (great-) grandparents of the 19th and early 20th century managed to rediscover geometry, starting from objects named metric spaces [4]. And although metric spaces might start off as too abstract, they can become beautiful and special enough to save mathematicians in need if given particular properties. CAT(0) cube complexes are among such spaces, and Petra Schwer's introductory book is a welcome (and perhaps needed) addition to the libraries of enthusiasts and newcomers alike.

Explaining the 'four elements' that opened the first paragraph, CAT(0) cube complexes are — as their name suggests — polyhedral complexes built out of (Euclidean) cubes glued in a natural way along faces, endowed with the induced length metric. The 'CAT(0)' bit means that triangles in the given space satisfy the Cartan–Alexandrov–Topogonov condition of being 'at most as thick' as flat (i.e., Euclidean) triangles. What further sets CAT(0) cube complexes apart is the existence of hyperplanes, which not only divide the complex into half-spaces but can be used to reconstruct the whole complex.

Since Gromov's groundbreaking essay [3] and Bridson–Haefliger's influential book [2], CAT(0)-ness became arguably the most popular notion of non-positive curvature, generalising to geodesic metric spaces the familiar concept of Riemannian manifolds having non-negative sectional curvature. The main point here, as Schwer rightfully reminds us in her text, is that those CAT(0) spaces which can be cubulated enjoy a plethora of nice geometric and combinatorial properties. For group theorists, this means that groups acting geometrically on such spaces also have many useful properties. Interestingly, CAT(0) cube complexes do arise in further contexts also of importance to our fellow scientists from other disciplines; the reader is referred to the excellent expository paper by Federico Ardila-Mantilla about CAT(0) cube complexes and the role of mathematics in society [1].

Petra Schwer's book is one of those that successfully accomplishes two tasks: introducing newcomers to a modern research area, and giving experts a reliable source of important known results. While the book starts with an understandable group-theoretic mindset, it is geometric in its core: needed basic concepts and tools about geodesic and CAT(0) spaces are given in the first half of Chapter 2 and in Chapter 3 — highly influenced by Bridson–Haefliger [2] — while Chapters 4 and 5 develop the basic theory about CAT(0) cube complexes, which so far was not yet done at a textbook level. This includes a thorough treatment of half-space structures and Rollar duality, and a full proof of Sageev's cubulation method, updated by the author's own input and previous developments in the literature. From Chapters 2 to 6, groups show up mostly as a source of examples for cube complexes, namely via Salvetti complexes of right-angled Artin groups, and the cubulation method for Coxeter groups, which have a dedicated chapter due to their importance and the author's expertise. Those interested in (more) group theory, 3-manifolds, and trees are taken to a well-documented tour in the final panoramic chapter. A further highlight, perhaps as important as the content itself, is the amount of carefully produced illustrations backing up concepts, arguments, and examples.

Though where Schwer's book shines the most in the reviewer's opinion — who very much enjoys teaching — is as a textbook for lectures. This might seem unsurprising given that the book grew out of Schwer's own lecture notes on the topic, taught a few times across Germany. However, it is not uncommon for lecture notes to grow too much or lose some focus when becoming a textbook, particularly for those dealing with very active research topics. Schwer's book does not fall into those traps and is kept to a reasonable length while being fairly self-contained. Proofs and examples are easily usable or adaptable for a one-semester lecture focusing on the geometry, on groups, or on both — depending on background and level of the students. And thankfully the author makes our artistic lives easier by providing the above-mentioned well-crafted pictures, which are fun to copy on a blackboard.

Overall the reviewer highly recommends Schwer's book to anyone interested in cube complexes and related areas, and commends the overall structure of the text and how well it reads. The book serves equally well students and researchers wanting to know more about what makes CAT(0) cube complexes so special (pun intended!).

FURTHER READING

[1] F. Ardila-Mantilla, CAT(0) Geometry, Robots, and Society, Notices Amer. Math. Soc. 67 (2020), no. 7, pp. 977–987.
[2] M. Bridson, A. Haefliger, Metric spaces of non-positive curvature, Grundlehren math. Wiss. 319, Springer, 1999.
[3] M. Gromov, Hyperbolic groups, in: Essays in group theory, Math. Sci. Res. Inst. Publ. 8 (1987), pp. 75–263.
[4] B. A. Rosenfeld, A history of non-Euclidean geometry, Springer, 1988.

Yuri Santos Rego

Dr Yuri Santos Rego is a lecturer in mathematics at the University of Lincoln. His research interests lie in anything related to groups – particularly topologically or cohomologically flavoured. Yuri hails from a Brazilian island, Upaon-Açu, where there is only one (awesome) season: burning hot. He is a big fan of Metroid (the video game series, not the subclass of matroids) and enjoys cooking with his family.

Obituaries of Members

Frederick C. Piper: 1940–2024

Professor Frederick C. Piper, who was elected a Member of the London Mathematical Society on 17 January 1963, died on 11 March 2024, aged 83, after a long illness.

Chris Mitchell writes: Known to everyone simply as Fred, he took his BSc and PhD at Imperial College (the latter under the supervision of Otto Wagner), before taking up a lectureship at Royal Holloway in the mid-1960s. He moved on to Westfield College, University of London a few years later, where he became a Reader and then Professor in Mathematics. His wonderfully clear and inspiring lectures made an enduring impression on all those of us fortunate to be taught by him.

At Westfield, Fred joined the late Dan Hughes, and they together formed a formidable research partnership, enduring through the 1970s and beyond. They co-authored a milestone book *Projective Planes*, which became a standard graduate level textbook. Fred moved back to Royal Holloway in the mid-1980s and developed a research group in Cryptography, working closely with Peter Wild. Fred's interest in cryptography had been sparked by a former PhD student of his, Henry Beker, who in the late 1970s worked at Racal Datacom; Fred and Henry jointly published a trailblazing book on cryptography, *Cipher Systems*, in 1982.

Fred served as Head of Department of Mathematics at both Westfield and Royal Holloway. By the early 1990s Fred was leading a research group in Information Security, spanning the Mathematics and Computer Science Departments, and with interests much broader than just cryptography. Under his guidance, the Information Security Group (ISG) launched an innovative MSc in Information Security in 1992 — probably the first degree of this type anywhere. Today the ISG is an academic department in its own right, and its ongoing success is a testament to Fred's dedication and energy. Over subsequent years Fred's own interests broadened to the industry and commerce focus of information security, where he championed early efforts to develop the security profession through The Institute of Information Security Professionals

Fred cared deeply about his PhD children, and was always keen to work with his ex-students. Many former research students of Fred have gone on to develop careers in security. For example the late Mike Walker, Fred's second PhD student, became a world-renowned expert on Cyber and Mobile Security and Cryptography serving as R&D Director for the Vodafone Group worldwide. Klaus Vedder became Group Senior Vice President at Giesecke and Devrient.

Not only did Fred make great advances in research but just as important he caused research and development to happen. At a time when it was not yet usual for industry and academia to work together, Fred coined the term 'academia and industry in harmony'. He encouraged research and development professionals in industry to work with academia, not by 'selling' projects to them but by facilitating collaboration. He organised events that would interest professionals in industry, listened to the requirements of industry, involved industry in research and shaped the way teaching and research was done to allow academics and industrial professionals to work together. Fred has influenced and supported industry in a manner that few could claim.

Fred received many awards, including the IMA Gold Medal in 2002. He led the ISG to the Queen's Anniversary Prize in 1998. He gave the 2004 BCS/IEE Turing Lecture. He was awarded an honorary CISSP in 2002, the first European to be given such an honour, and an honorary CISM in 2003. He was named Network Professional of the Year in the 2005 Communications in Business Awards, and was added to the ISSA Hall of Fame in 2005. In 2008 he was elected as a Fellow of (ISC)2, and he was the first person to be elected to the InfoSecurity Europe Hall of Fame. In 2011 he was awarded an Honorary Fellowship by Royal Holloway, University of London. In 2021 the CIISec named an annual award after him (the Fred Piper Award). Fred was held in the highest of esteem worldwide.

Above all else, I think Fred would be happiest with the knowledge that through his teaching, wise counsel and encouragement, he personally touched — and improved — the lives of so many.

LMS Meeting

LMS Popular Lecture 2024

Thursday 9 May 2024 (18:30 – 20:15), University of Birmingham, Bramall Music Building

Website: lms.ac.uk/events/lms-popular-lecture-2024

The LMS Popular Lecture will this year be given by Sarah Hart (Gresham Professor of Geometry, Professor Emerita of Mathematics and Fellow of Birkbeck, University of London), author of *Once Upon A Prime: The Wondrous Connections between Mathematics and Literature*. The talk will be titled *The beautiful connections between mathematics and literature*. Popular Lectures are free public talks which present exciting topics in mathematics and its applications to a wide audience. They are for anyone with an interest in mathematics, including those without high-level maths training.

Wales Mathematics Colloquium 2024

Location: Gregynog Hall, Powys
Date: 20-22 May 2024
Website: gregynogwmc.github.io

The Colloquium brings together mathematicians of all disciplines from across Wales and beyond. Alongside contributed talks are lectures from guest speakers: Professor Geoffrey Grimmett (probability), Professor Kevin Buzzard (pure maths) and Professor Julia Gog (mathematical biology). Research students are particularly encouraged to attend and deliver talks. There will also be a session delivered in Welsh. Register via the website by 12 April 2024.

Identifying Threshold Concepts in Undergraduate Mathematics

Location: Queen Mary University of London
Date: 30 May 2024
Website: tinyurl.com/fphas53m

A one day in-person workshop on identifying and examining the implications of threshold concepts in undergraduate mathematics teaching, which will be of interest to academic lecturers and others working in in higher education. It will provide a valuable opportunity for cross-institutional discussions about teaching and learning in mathematics. The event is free to register, luncheon, tea and coffee refreshments will be provided. Attendance is limited to the first 50 participants, deadline to register being 24 May 2024. This event is partially funded by an LMS Grant for Teaching and Learning in HE.

LMS Education Day

Location: De Morgan House, London
Date: 22 May 2024
Website: lms.ac.uk/events/education-day-2024

The annual LMS Education Day, organised by the LMS Education Committee, is an opportunity for mathematics lecturers and colleagues across the higher education sector to meet and discuss aspects of education directly related to higher education. It is a free in-person event held at De Morgan House, London. This year's Education Day will be on *Mathematics, AI and Digitech Futures*. See the website for a full list of speakers and to register.

Algebraic and Geometric Aspects of Differential and Difference Equations

Location: University of Portsmouth
Date: 10-13 June 2024
Website: smap.port.ac.uk/AGADDE

The aim of workshop is to bring together experts in the field of differential and difference equations that give rise to solutions that are special functions, Painlevé transcendents and more general ones. One particular aspect is the algebraic and geometric theory of such equations, in connection with their Okamoto's spaces of initial conditions. This sits in the broader context of integrable mappings. Please register on the website by the end of May. This event is partially funded by an LMS Conference Grant

Research Students' Conference in Population Genomics

Location: University of Edinburgh
Date: 26-28 June 2024
Website: tinyurl.com/4swz8sxk

The Research Students' Conference in Population Genomics 2024 intends to address every facet of this emerging field through interdisciplinary collaboration with three themes: evolutionary networks, causal effect estimation and pathogen genetics. It is the culmination of the developing relationship between big data, mathematics and biological understanding. This event is partially funded by an LMS Scheme 8 Postgraduate Research Conference Grant.

LMS General Meeting and Celebration of Kelvin's 200th Anniversary

Location: De Morgan House, London
Date: 28 June 2024
Website: tinyurl.com/3f5uyrtt

The first lecture will be given by Mark McCartney (Ulster University) with accompanying lectures by Ruiping Mu (Northwest University), Luke K. Davis (University College London), Rosalba Garcia-Millan (Kings College London), Jemma Lorenat (Pitzer College) and Joe Goddard (UC San Diego). The meeting is held in partnership with the British Society of the History of Mathematical and the University of Glasgow.

LMS Meeting

LMS Invited Lecture Series 2024

1-5 July 2024, Imperial College, London

Website: sites.google.com/view/lms-invited-lectures-2024

LMS Invited Lecturer 2024: Dan Abramovich (Brown University)

Title of talk: *Logs and Stacks in Birational Geometry and Moduli*

Accompanying lectures by: Hülya Argüz (Georgia), Pierrick Bousseau (Georgia), Francesca Carocci (Lausanne), Navid Nabijou (Queen Mary) and Dhruv Ranganathan (Cambridge).

For further details about the LMS Invited Lecture Series and to register for a place, please visit sites.google.com/view/lms-invited-lectures-2024.

Funds are available for partial support to attend the LMS Invited Lecture Series. Requests for support with an estimate of expenses should be addressed to the organisers Alessio Corti (a.corti@imperial.ac.uk).

The biennial Invited Lecturers Series aim to bring a distinguished overseas mathematician to the United Kingdom to present a small course of about ten lectures spread over a week. Each course of Invited Lectures is on a major field of current mathematical research, and is instructional in nature, being directed both at graduate students beginning research and at established mathematicians who wish to learn about a field outside their own research specialism.

Trending Aspects in Group Cohomology

Location: University of Lincoln
Date: 2-3 July 2024
Website: tinyurl.com/485w8zzh

This mini-workshop focuses on recent developments in the cohomology theory of infinite groups. The event is supported by an LMS Scheme 9 Grant (Celebrating New Appointments). The invited speakers are Ilaria Castellano (Bielefeld), Dawid Kielak (Oxford) and Brita Nucinkis (London). Some limited funding to support UK-based participants is available. Postgraduate students and young postdocs are particularly encouraged to register. Further information, including the registration form, can be found on the website above.

DANE 2024: Design and Analysis of Networked Experiments

Location: King's College London
Date: 17-19 July 2024
Website: tinyurl.com/32hudx52

Design and analysis of networked experiments is a novel area of research which finds practical use in most research disciplines, medical trials and industrial settings that involve connected experimental units. The aim of this workshop is to enable in-depth discussions and initiate synergies between two dominant philosophies in networked experiments, optimal design and causal inference. It is open to researchers and practitioners in the area of design and analysis of experiments with interests in experiments on networks. This workshop is supported by an LMS Conference Grant.

Young Mathematicians in C*-algebras 2024

Location: University of Glasgow
Date: 5-9 August 2024
Website: sites.google.com/view/ymca-2024

The Young Mathematicians in C*-Algebras (YMC*A) is an annual conference to bring together early-career researchers in the field of operator algebras. It provides opportunities for researchers to share their work and learn from more experienced plenary speakers. This conference will also feature a ceilidh (Scottish dance party), and various excursions around Glasgow. Deadline to register is 1 June. Deadline to submit a talk is 1 June.

30th British Combinatorial Conference

Location: Queen Mary University of London
Date: 1-5 July 2024
Website: tinyurl.com/2s3v9r9j

The British Combinatorial Conference is the foremost UK conference on combinatorics and its applications. In 2024 it will include nine plenary talks accompanying articles published in a volume of Surveys in Combinatorics, six mini-symposia covering the breadth of combinatorics and numerous contributed talks. The deadlines for submission of contributed talks and registration are 5 May and 17 June 2024. The conference is supported by an LMS Conference Grant.

AGGITaTE 2024: LMS/HIMR Research School

Location: University of Essex
Date: 22-26 July 2024
Website: claymath.org/events/aggitate-2024

This school will introduce developments in the theory of moduli spaces to postgraduate research students and early-career researchers. It is organised around three courses, each of which presents a different perspective on the study of moduli spaces, and addresses different problems. The school will give early career researchers the opportunity to learn from the leaders in the field themselves, and to start research in this area. Closing date for applications: 1 May.

Geometry from the Model Theorist's Point of View

Location: University of Oxford
Date: 10-13 September 2024
Website: people.maths.ox.ac.uk/bays/Z75/

A conference arranged in honour of Boris Zilber's 75th birthday, concentrating on manifestations of geometric ideas in model theory. Speakers include John Baldwin, Zoé Chatzidakis, Francesco Gallinaro, Assaf Hasson, Åsa Hirvonen, Ehud Hrushovski, Vincenzo Mantova, Rahim Moosa, Kobi Peterzil, Anand Pillay, Bruno Poizat, Thomas Scanlon, Katrin Tent, and Boris Zilber. This event is partially funded by an LMS Conference Grant.

Society Meetings and Events

May

9 LMS Popular Lecture 2024, Speaker: Sarah Hart, Birmingham

22 LMS Education Day 2024, London

June

19 Society Meeting at the BMC. Speaker: Corinna Ulcigrai (Zürich))

26-28 Research Students' Conference in Population Genomics

26-30 LMS Research School Random Structures, Applied Probability and Computation, Liverpool

28 LMS General Meeting and Kelvin 200th Anniversary Lecture, London

July

1-5 LMS Invited Lecture Series 2024, Imperial College London

3-7 LMS Research School Machine Learning in Mathematics and Theoretical Physics, Oxford

16-28 LMS Undergraduate Summer School, Sheffield

19-23 LMS Invited Lecture Series 2023, Durham

24-28 LMS Research School Algebraic Groups and their Representations, Birmingham

24 – 4 August LMS-Bath Mathematical Symposium Operators, Asymptotics, Waves, hosted at the University of Bath

August

1-11 LMS-Bath Mathematical Symposium Categorical and Geometric Representation Theory, University of Bath

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

May

20-22 Wales Mathematics Colloquium 2024, Gregynog Hall, Powys

30 Identifying Threshold Concepts In Undergraduate Mathematics, Queen Mary University of London

June

10-13 Algebraic and Geometric Aspects of Differential and Difference Equations, University of Portsmouth

12-14 UK Operator Algebras Conference, Newcastle University

17-20 British Mathematical Colloquium 2024

24-28 The Interplay of Geometric Group Theory and K-Theory, University of Southampton

26-28 Population Genomics Research Students' Conference, University of Edinburgh

July

2-3 Trending Aspects in Group Cohomology Mini-Workshop, University of Lincoln

2-4 OPSFOTA: Orthogonal Polynomials, Special Functions, Operator Theory and Applications, University of Bristol

1-5 30th British Combinatorial Conference, Queen Mary University of London

8-12 British Isles Graduate Workshop V: Mathematical General Relativity, Coalport, Shropshire

15-19 2024 European Congress of Mathematicians, Seville, Spain

8-12 Simple-Mindedness, Silting, and Stability, University of Cumbria, Ambleside

17-19 DANE 2024, King's College London

22-26 AGGITATE 2024, University of Essex

August

5-9 Young Mathematicians in C*-algebras 2024, University of Glasgow

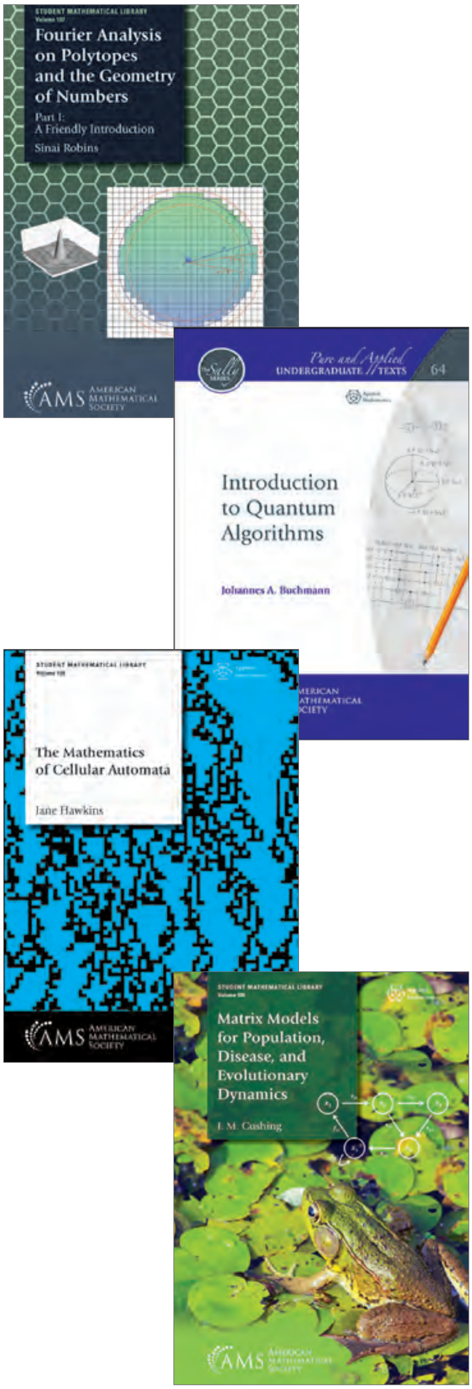
12-16 International Workshop on Operator Theory and its Applications (IWOTA)

September

4-6 Groups & Representations: after Roger Carter, University of Warwick

10-13 Geometry from the Model Theorist's Point of View, Math Institute & St. Hilda's College, Oxford

AMERICAN MATHEMATICAL SOCIETY



FOURIER ANALYSIS ON POLYTOPES AND THE GEOMETRY OF NUMBERS

Part I: A Friendly Introduction

Sinai Robins, University of São Paulo

Student Mathematical Library, Vol. 107

A gentle introduction to the geometry of numbers from a modern Fourier-analytic point of view. One of the main themes is the transfer of geometric knowledge of a polytope to analytic knowledge of its Fourier transform. The Fourier transform preserves all of the information of a polytope, and turns its geometry into analysis.

May 2024 349pp 9781470470333 Paperback £52.00

INTRODUCTION TO QUANTUM ALGORITHMS

Johannes A. Buchmann, Technical University of Darmstadt

Pure and Applied Undergraduate Texts, Vol. 64

The theory of quantum algorithms is based on advanced concepts from computer science, mathematics, and physics. This book offers a mathematically precise exploration of these concepts, accessible to those with a basic mathematical university education, while also catering to more experienced readers.

Apr 2024 371pp 9781470473983 Paperback £78.00

THE MATHEMATICS OF CELLULAR AUTOMATA

Jane Hawkins, University of North Carolina at Chapel Hill

Student Mathematical Library, Vol. 108

A rigorous mathematical introduction to cellular automata (CA). Numerous colourful graphics illustrate the many intriguing phenomena, inviting undergraduates to step into the rich field of symbolic dynamics. Exercises and labs end each chapter, covering a range of applications, both mathematical and physical.

Apr 2024 228pp 9781470475376 Paperback £52.00

MATRIX MODELS FOR POPULATION, DISEASE, AND EVOLUTIONARY DYNAMICS

J. M. Cushing, University of Arizona

Student Mathematical Library, Vol. 106

An introduction to the use of matrix theory and linear algebra in modeling the dynamics of biological populations. Learn the basics of modeling methodology (i.e., how to set up a matrix model from biological underpinnings) and the fundamentals of the analysis of discrete time dynamical systems (equilibria, stability, bifurcations, etc.).

Mar 2024 305pp 9781470473341 Paperback £52.00

Order online at **eurospanbookstore.com/ams**

CUSTOMER SERVICES

Individual Customers:
Tel: +44 (0)1235 465577
direct.orders@marston.co.uk

Trade/Account Customers:
Tel: +44 (0)1235 465576
trade.orders@marston.co.uk

FURTHER INFORMATION
Tel: +44 (0)20 7240 0856
info@eurospan.co.uk

AMS is distributed by **EUROSPAN**