LONDON
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NEWSLETTER

Issue: 483 - July 2019

50 YEARS OF THERMODYNAMIC FORMALISM

MATHEMATICS AND MINE DETECTION

EXACT AND APPROXIMATE COMPUTATIONS

EDITORIAL OFFICE

London Mathematical Society
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COVER IMAGE

The cover shows part of the demining process (see the feature starting page 27): after a positive signal on a metal detector, a thin metal rod us used to probe carefully to see if there is a mine. (Image courtesy of Bill Lionheart.)

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The Newsletter is freely available electronically at lms.ac.uk/publications/lms-newsletter.

MEMBERSHIP

Joining the LMS is a straightforward process. For membership details see lms.ac.uk/membership.

SUBMISSIONS

The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at iain.moffatt@rhul.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see lms.ac.uk/publications/advertise-in-the-lms-newsletter.

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Change of date for the AGM

The Society would like to draw Members' attention to the change of date for the Annual General Meeting from Friday 15 November 2019 (as previously advertised) to Friday 29 November 2019. The change of date is to ensure that there is sufficient time for the Society to fulfil its obligations, according to the timetable set by the Privy Council, with respect to the proposal to change aspects of the Society's current Standing Orders that form part of its Royal Charter. Further information about the LMS Standing Orders Review can be found on the Society's website here: lms.ac.uk/about/lms-standing-orders-review.

Fiona Nixon
Executive Secretary

Reciprocity Agreement with the Irish Mathematical Society

The London Mathematical Society (LMS) is delighted to announce that, from 1 July 2019, the Society will have a Reciprocity Agreement in place with the Irish Mathematical Society (IMS).

LMS Members who are not normally resident in the Republic of Ireland can now join, or renew their subscription to, the Irish Mathematical Society and pay half the normal membership fee. LMS Members who are not normally resident in the United Kingdom, and who are also members of the Irish Mathematical Society, can now request a change in their membership type to Reciprocity member by emailing membership@lms.ac.uk.

Council has also agreed that those who are both LMS and IMS members and are resident in Northern Ireland may choose to which Society they wish to pay the full membership fee and to which Society they would wish to pay the Reciprocity (half-price) fee. If such members wish to request a change in their membership type to Reciprocity member, please email membership@lms.ac.uk.

LMS Reciprocity membership fees are one half of the Ordinary membership fee e.g. £42.00 compared with £84.00 for 2018–19. Changes to membership fees are not retrospective and we regret that a refund cannot be given on fees already paid.

Email membership@lms.ac.uk with any membership queries. See page 36 for more information on the Irish Mathematical Society.

Robert Curtis
LMS Treasurer

Non-plastic *Newsletter* Wrappers

Readers may have noticed that since March, the *Newsletter* has been sent out in 100% compostable potato starch wrapping, rather than in plastic. Please dispose of your wrappers in a compost heap, garden waste bin, or a food waste bin.

New Editor-in-Chief sought for the *Newsletter*

The LMS is currently seeking someone to take over when the current Editor-in-Chief, Iain Moffatt, stands down at the end of the year.

Stephen Huggett (LMS General Secretary) writes: Iain has provided invaluable leadership and guidance to the current Editorial Board for the last three years. His appointment followed a review of the *Newsletter* in 2016. Iain and the Board have delivered a refreshed and revived Newsletter which now includes mathematics articles and features and a section for early career researchers, as well as reporting on Society business and news for the membership and further afield. Iain has also been instrumental in introducing a more professional format to the *Newsletter* and in ensuring that its production is undertaken to the same exacting standards as the process for commissioning content. Many favourable comments have been received about the current *Newsletter* and in particular the inclusion of mathematics into the format as well as the quality of the content. The Society would like to take this opportunity publicly to thank Iain for his dedication to producing such a successful publication and for the amount of time he has given to ensuring that the Society has such a high quality *Newsletter*.

The Editor-in-Chief is responsible to the Council of the LMS for chairing the Newsletter Editorial Board. The main duties of the role are: overseeing the commissioning of content for the mathematical features section, overseeing the sourcing of material for other sections, signing off on the content and layout for each bi-monthly issue, chairing the Newsletter Editorial Board which meets two times per year to discuss both ongoing editorial and production issues, and wider strategic matters.

Appropriate candidates may have some of the following attributes: an interest in the Society and its work, editorial experience, the diplomatic skills required to ensure that articles are continuously commissioned and submitted to provide adequate content for each issue of the *Newsletter*, some experience of LaTeX, copyediting and production, a good eye for design, an understanding of the need to include content which is of interest to a broad constituency of readers across a wide range of subjects, not just with regard to mathematics but also of wider issues which might affect mathematics and mathematicians such as policy matters and the Society's international connections, a knowledge of the Society's activities. Above all, candidates must be prepared to commit the time needed to ensure sufficient content and manage the practicalities of the production of each issue of the *Newsletter*.

It should be noted that this is an unremunerated role, although reasonable expenses will be paid.

Applications, comprising a CV and a statement on how your experience might fit with the attributes listed above and what you might bring to the role, should be sent to the Executive Secretary at fiona.nixon@lms.ac.uk using the subject line: Editor-in-Chief application, no later than **Friday 12 July 2019**.

It is intended that interviews will take place in the week beginning 22 July 2019 with a view to the successful candidate shadowing the current Editor-in-Chief from September/October this year to follow the full production cycle for the January 2020 issue of the *Newsletter*.

Any person interested in submitting an application, who wishes to discuss the role further may contact the current Editor-in-Chief, Iain Moffatt, by emailing in the first instance to Iain.Moffatt@rhul.ac.uk.

Stephen Huggett
General Secretary

LMS Invited Lecture Series 2020

In spring 2020 Professor Yuliya Mishura will visit the Mathematical Department of Brunel University London, and give a condensed course on fractional calculus and fractional stochastic calculus, including rough-paths, and applications.

Professor Mishura has being working at Taras Shevchenko National University of Kyiv, Ukraine since 1976, becoming a professor in 1991. She has been the head of the Department of Probability, Statistics and Actuarial Mathematics since 2003. During her career her work has evolved from the theory of random fields, including martingale and semimartingale methods, to stochastic analysis and statistics of fractional processes.

At present Yuliya Mishura is one of the most recognised experts in the theory of fractional processes. She is the author of more than 260 publications, including nine books. Her monograph *Stochastic Calculus for Fractional Brownian Motion and Related Processes* (2008) is a very popular textbook among the beginners and specialists in the field of fractional Gaussian processes.

The course will be held from 30 March to 3 April 2020, over a total of 10 hours of lectures. This course is aimed at a broad mathematical audience interested in working with long- and short-memory processes. In particular it will be beneficial for PhD students. The course will be accompanied by exercise sessions.

The course will highlight the interplay between fractional stochastic processes and differential equations with respect to fractional operators, where random processes arise. The random processes which arise from solving fractional differential equations are very different from fractional stochastic processes. Luckily the fractional calculus happened to be the connecting bridge between them. The purpose of the proposed course is to present both an exposition of the basic ideas in the theory of fractional stochastic processes and the main ideas from the theory of differential equations with fractional operators. Both these areas at the moment are very popular and are rapidly developing, speeded up by the large number of applications.

The topics will include elements of fractional calculus, fractional Gaussian processes and their main properties, elements of rough-path theory, partial stochastic differential equations involving fractional processes, and various applications. The required background is masters-level probability, familiarity with differential equation theory is also useful.

Contact the local organiser, Elena Boguslavskaya, for more details: Elena.Boguslavskaya@brunel.ac.uk.

LMS–NZMS Aitken Lecture Tour

The Aitken Lectureship occurs every two years (in odd-numbered years) when a mathematician from New Zealand is invited by both the London Mathematical Society and the New Zealand Mathematical Society to give lectures at different universities around the UK over a period of several weeks.

Professor Bakh Khoussainov is the fifth holder of the Aitken Lectureship. He is visiting the UK during June and July 2019. He gave the Aitken Lecture at the General Meeting of the Society on 28 June and an Aitken Lecture in Oxford on 26 June. He will give further Aitken Lectures at departments around the UK as follows:

- Durham (3 July) *Algorithmically random structures.* Local organiser: Iain A Stewart
- St. Andrews (8 July) *Semigroups, groups, algebras, and their finitely presented expansions.* Local organiser: Nik Ruskuc
- Liverpool (11 July) *Games played on finite graphs.* Local organiser: Frank Wolter
- Manchester (12 July) *Open questions on automatic structures.* Local organiser: Tuomas Sahlsten
- Swansea (16 July) *Semigroups, groups, algebras, and their finitely presented expansions.* Local organiser: John Tucker
- London Logic Colloquium at Birkbeck (17 July) *Games played on finite graphs.* Local organiser: Michael Zakharyaschev

For further details on how to attend these lectures, please contact the named local organiser at each venue.

Further details about the LMS-NZMS Aitken Lecture Tour and the LMS-NZMS Forder Lecture Tour to New Zealand may be found at tinyurl.com/yxoz73ba.

DeMorgan@21

The opening of De Morgan House: (l to r) Richard Borcherds, Klaus Roth, Dan Quillen, Timothy Gowers, Michael Atiyah, Simon Donaldson, Alan Baker

Extract from the Council Minutes: Friday 16 October 1998:

> The President asked the Executive Secretary to report on the arrangements for the opening of De Morgan House. About 115 members and guests were expected for the opening on Friday 23 October. They would be received in the Lobby/Reception area, the Verblunsky room and the Council room. The opening would be transmitted by closed-circuit television. A reception would be held after the opening...

As announced in the January Newsletter, the 21st anniversary of this event, so momentous in the life of the LMS, will be celebrated on Saturday 19 October 2019. The event will take place in De Morgan House from 2–6 pm with talks, tea, cake and champagne. All LMS members are invited, although registration is necessary since as on the previous occasion, numbers will be somewhat limited owing to the capacity of De Morgan House.

As memorialised on the plaque in the entryway, almost all the British Fields medallists were present at the opening. Our present Fields medallists have all been invited and the most recent among them, Caucher Birkar, will be giving a short speech. There will also be a short talk on Augustus De Morgan by June Barrow-Green and memories from those, including Sir John Ball and Susan Oakes, involved in the purchase and move. We will also be reflecting on just how far the LMS has been able to develop in its new premises.

The organising committee has been searching the archives and collecting reminiscences and photos which will be displayed and gathered in a short booklet. Please send any further thoughts or material to Susan Oakes (Susan.Oakes@lms.ac.uk) as soon as possible. Further details about the programme and instructions on how to register will be available on the LMS website in late July. We look forward to seeing you there.

DeMorgan@21 Organising Committee:
June Barrow-Green, Susan Oakes,
Caroline Series, Sarah Zerbes

Forthcoming LMS Events

The following events will take place in the next three months:

Random Structures: from the Discrete to the Continuous, LMS Research School: 1-5 July, Bath (tinyurl.com/y8mn422e)

Mathematics of Climate, LMS Research School: 8–12 July, Reading (tinyurl.com/yb2ujfxk)

Prospects in Mathematics Meeting: 6–7 September, Lancaster (tinyurl.com/y4c9aaxk)

Midlands Regional Meeting: 11 September, Nottingham (tinyurl.com/y5vtaytx)

Popular Lectures: 19 September, University of Birmingham (tinyurl.com/hu58wjk)

A full listing of forthcoming LMS events can be found on page 54.

OTHER NEWS

Birthday Honours 2019

Awards in the Queen's Birthday honours that are related to mathematics include:

- Arnold Black (Historian and Statistician, Scottish Athletics) awarded an MBE for services to Athletics (for his work as a statistician with Scottish Athletics).
- Ken Brown (University of Glasgow) an LMS member and former Vice-President, awarded a CBE for his services to the Mathematical Sciences.
- Peter Donnelly (University of Oxford) an LMS member, awarded a knighthood for services to the Understanding of Human Genetics in Disease.
- Duncan Lawson (Coventry University) awarded an MBE for his services to Mathematics in Higher Education.
- Max Parmar (Professor of Medical Statistics and Epidemiology and Director, MRC Clinical Trials Unit, University College London) awarded an OBE for services to Medical Research and Clinical Trials.
- Peter Ransom (former MA President), an LMS member, awarded an MBE for his voluntary service to Mathematics Education.
- Sylvia Richardson (Director, Medical Research Council Biostatistics Unit, University of Cambridge) awarded a CBE for services to Medical Statistics.

Mathematicians Honoured by the Royal Society

The Fellowship of the Royal Society is made up of the most eminent scientists, engineers and technologists from or living and working in the UK and the Commonwealth. This year fifty-one new Fellows have been elected.

The seven elected mathematicians are Manjul Bhargava (Princeton University), Caucher Birkar (University of Cambridge), Christopher Hacon (University of Utah), Peter Haynes (University of Cambridge), Richard Jozsa (University of Cambridge), Roy Kerr (University of Canterbury) and Akshay Venkatesh (Institute of Advanced Studies).

The full list of new Royal Society Fellows is available at tinyurl.com/y5sfljeb.

MATHEMATICS POLICY DIGEST

UKRI Open Access Review

As part of the ongoing Open Access Review, UK Research and Innovation (UKRI) will welcome input from across the sector later this year through a public consultation. The UKRI Open Access Review concerns open access to formal scholarly research articles, peer reviewed conference proceedings and monographs. It is an opportunity to align policies across UKRI's councils, with the UK Funding Bodies on future Research Excellence Framework (REF) policy, and to consider how Innovate UK should be included. The current Research Councils UK (RCUK) policy continues to apply over the period of the review. There will be no change to the REF 2021 open access policy.

The Open Access Review is being undertaken in four phases of work, which started in autumn 2018 and will run to spring 2020. Throughout the Review UKRI will be engaging with a range of relevant stakeholders, and a public consultation on the draft UKRI policy will now take place later this year to allow further time for input from across the sector on the draft policy. The review is expected to report in March 2020 and UKRI expects the revised policy to apply during 2020. The LMS will be responding to this consultation. Further information is available at tinyurl.com/yy7cseay.

Science research funding in universities

The House of Lords Science and Technology Select Committee has opened a new inquiry to assess the impact of potential changes in funding on the ability of UK universities to conduct high-quality scientific research. More information is available at tinyurl.com/y447ndca.

Payments for early-career mathematics teachers

Eligible mathematics teachers can apply for early-career payments of up to £7,500, after tax, with a pilot scheme announced by the Department for Education. Details of the scheme and eligibility criteria are available at tinyurl.com/y3frd39b.

Digest prepared by Dr John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

EUROPEAN MATHEMATICAL SOCIETY NEWS

From the EMS President

This is a short summary of my vision for the EMS in the next four years.

Although the EMS is a well-functioning society, there is always room for improvement and there are major challenges on the horizon. We have to overcome the large imbalance of mathematical research and education in Europe. We should communicate better between the different fields of mathematics and with other sciences and we should strongly work together in lobbying for the advancement of mathematics in Europe. The different areas of mathematics should treat each other with higher respect than often in the past and we should get more active in the advancement of young mathematicians and to increase the visibility of mathematics in society and our young colleagues, in particular.

Mathematical publications are an important factor of our activities. In view of the fact that publishing becomes more and more electronic, the current publishing models will have to be rethought, and this will be a major activity for the EMS in the coming years. This also concerns the current open access strategies of the EU and the member states that will have a major effect on the way mathematical publishing in the future will look.

Finally, many of us are complaining about the fact that mathematics education is getting worse in Europe. The whole community has to make major efforts to avoid further degradation of the standards and to improve math education as a whole but also for highly talented young people.

I am looking forward to four years of working together with you for the well-being of mathematics in Europe.

Volker Mehrmann
President of the EMS

Impact of Mathematics

Members of the LMS are likely to be aware of the Deloitte report *Measuring the Economic Benefits of Mathematical Science Research in the UK* published by EPSRC and CMS in 2013. Unsurprisingly, several other European countries have also released reports on the impact of mathematics in society: Socio-economic impact of mathematical research and mathematical technology in Spain (tinyurl.com/y6gwo3ox), Etude de l'impact socio-économique des Mathématiques en France (tinyurl.com/y24myt69), Formulas for Insight and Innovation: Mathematical Sciences in the Netherlands (tinyurl.com/y3nvaq75).

Arrest of Tuna Altinel

On 11 May 2019 Tuna Altinel, a Turkish mathematician working for many years in the Université Lyon 1 in France, was arrested in Turkey. Professor Altinel was one of the signatories in 2016 of a petition supported by more than 2000 scientists and intellectuals against military actions towards civilians, and took part in a legitimate public meeting in France in February 2019 on the same topic.

The LMS and the EMS condemn this violation of Professor Altinel's human rights. More information is available on tinyurl.com/y64ts42x.

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

OPPORTUNITIES

Ferran Sunyer i Balaguer Prize: call for nominations

The prize will be awarded for a mathematical monograph of an expository nature presenting the latest developments in an active area of research in mathematics. The monograph must be original, unpublished and not subject to any previous commitment edition. The prize consists of €15,000 and the winning monograph will be published in the Birkhäuser series *Progress in Mathematics*.

Deadline for submission is 29 November 2019. For further information visit the website at ffsb.iec.cat.

Stephen Smale Prize: call for Nominations

The fourth Stephen Smale Prize will be awarded at the Foundations of Computational Mathematics (FoCM) Meeting in Vancouver, Canada, 15–24 June 2020. The goal of the Smale Prize is to recognize major achievements in furthering the understanding of the connections between mathematics and computation, including the interfaces between pure and applied mathematics, numerical analysis, and computer science. To be eligible for the prize a candidate must be in his or her early to mid-career, meaning, typically, removed by at most ten years from his or her (first) doctoral degree by the first day of the FoCM meeting. Eligible candidates should be nominated by email to the FoCM Secretary Michael Singer (singer@ncsu.edu) no later than 15 September 2019.

The recipient of the prize will be expected to give a lecture at the meeting. For more information, see focm-society.org/smale_prize.php.

W.K. Clifford Prize: call for nominations

The W.K. Clifford Prize is an international scientific prize designed to encourage young researchers to compete for excellence in theoretical and applied Clifford algebras, their analysis and geometry. The award consists of a written certificate, one year of online access to two Clifford algebra related journals, a book token worth €150, and a cash award of €1,000. The laureate is also offered the opportunity to give the special W.K. Clifford Prize Lecture at University College London, where W.K. Clifford held the first Goldsmid Chair from 1871 until his death in 1879.

The next W.K. Clifford Prize will be awarded at the 12th Conference on Clifford Algebras and Their Applications in Mathematical Physics (ICCA12) at the

University of Science and Technology of China in 2020.

The deadline for nominations is 30 September 2019. Nominations should be sent to CliffordPrize2020@gmail.com. For details see wkcliffordprize.wordpress.com.

LMS Grant Schemes

For full details of these grant schemes, and for information on how to submit an application form, visit www.lms.ac.uk/content/research-grants.

RESEARCH GRANTS

The deadline is 15 September 2019 for applications for the following grants, to be considered by the Research Grants Committee at its October meeting.

Conferences Grants (Scheme 1): Grants of up to £7,000 are available to provide partial support for conferences held in the United Kingdom. This includes a maximum of £4,000 for principal speakers, £2,000 to support the attendance of research students who are studying at universities in the UK, and £1,000 to support the attendance of participants from Scheme 5 eligible countries.

Visits to the UK (Scheme 2): Grants of up to £1,500 are available to provide partial support for a visitor to the UK, who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. It is expected the host institutions will contribute to the costs of the visitor.

Joint Research Groups in the UK (Scheme 3): Grants of up to £4,000 are available to support joint research meetings held by mathematicians who have a common research interest and who wish to engage in collaborative activities, working in at least three different locations (of which at least two must be in the UK). Potential applicants should note that the grant award covers two years, and it is expected that a maximum of four meetings (or an equivalent level of activity) will be held per academic year.

Research in Pairs (Scheme 4): For those mathematicians inviting a collaborator to the UK, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research.

Collaborations with Developing Countries (Scheme 5): For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position.

Computer Science Small Grants (Scheme 7): Grants of up to £1000 are available to support visits for collaborative research at the interface of Mathematics and Computer Science, either by the grant holder to another institution within the UK or abroad, or by a named mathematician from within the UK or abroad to the home base of the grant holder.

Research Workshop Grants: Grants of between £3,000 - £5,000 are available to provide support for Research Workshops held in the United Kingdom, the Isle of Man and the Channel Islands.

AMMSI African Mathematics Millennium Science Initiative: Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS-AMMSI grants are available at ammsi.or.ke.

GRANTS FOR EARLY CAREER RESEARCHERS

The deadline is 15 October 2019 for applications for the following grants, to be considered by the Early Career Research Committee in November.

Postgraduate Research Conferences (Scheme 8): Grants of up to £4,000 are available to provide partial support for conferences held in the United Kingdom, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants.

Celebrating new appointments (Scheme 9): Grants of up to £600 are available to provide partial support for meetings held in the United Kingdom to celebrate the new appointment of a lecturer at a UK university.

Travel Grants for Early Career Researchers: Grants of up to £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

Spitalfields Day 2019–20: call for proposals

The London Mathematical Society offers funding of up to **£1,000** towards the cost of a Spitalfields Day. A Spitalfields Day is a one-day event at which selected participants, often eminent experts from overseas, give survey lectures or talks, which are accessible to a general mathematical audience. The Spitalfields Day is often associated with a long-term symposium and speakers will generally give lectures on topics of the symposium.

The name honours the Society's predecessor, the Spitalfields Mathematical Society, which flourished from 1717 to 1845, and Spitalfields Days have been held each year since 1987.

The funding of £1,000 is intended to cover actual supplementary costs for the event, e.g. subsidising the cost for a lunch for participants, and for small travel grants of £50 to enable LMS members and research students to attend the event.

If you are interested in organising a Spitalfields Day, please write to the Society (lmsmeetings@lms.ac.uk). The format need not be precisely as described, but should be in a similar spirit. The next deadline for proposals is **15 September 2019**. Subsequent deadline is 31 January 2020. Please note the Society cannot fund events retrospectively so applicants are advised to apply well in advance of the event.

Previously supported Spitalfields Days have included:

2015: York. *Mathematics of Quantum Uncertainty: New Advances and Prospects.* Speakers: Pekka Lahti (Turku, Finland), Reinhard Werner (Hannover, Germany), Madalin Guta (Nottingham) and Paul Busch (York).

2014: INI, Cambridge. *Theory of Water Waves.* Speakers: Mark Groves (Loughborough/Saarland), Guido Schneider (Stuttgart), Steve Shkoller (Oxford) and Eugene Varvaruca (Reading).

Invitation for IMU book bids

On the occasion of its 100th anniversary, the International Mathematical Union (IMU) invites potential authors to write a monograph (tinyurl.com/y5ngzcvw) presenting the IMU in a larger framework of international scientific unions, and the internationalization of science and mathematics, in particular. The Klaus Tschira Stiftung, Heidelberg, Germany, generously funds the project. Bids should be submitted in pdf by email to IMU Secretary General Helge Holden (secretary@mathunion.org) no later than 1 August 2019.

Nevanlinna and Abacus Prizes

Since 1982 the IMU has awarded the Rolf Nevanlinna Prize at each ICM for outstanding contributions in Mathematical Aspects of Information Sciences. In 2018, the IMU Executive Committee took the decision to discontinue this prize because of historical issues arising from its name.

It was decided at the IMU General Assembly 2018 to set up a new prize with similar scope. It will be called the IMU Abacus Medal and will be awarded at each ICM, starting in 2022. The award will consist of a gold medal, a cash award of €10,000, and a diploma.

Bids are now being invited to provide funding for both the medal and the cash award. Further details may be found at tinyurl.com/y3k28l7c. Bids should be submitted by 1 October 2019 to the IMU Secretary General Helge Holden at secretary@mathunion.org.

VISITS

Visit of Benjamin Steinberg

Professor Benjamin Steinberg (City College of New York) will visit the University of East Anglia from 15 to 21 September 2019. He is an algebraist interested in a broad range of areas including semigroups, geometric group theory, algebraic combinatorics and representation theory, automata theory, finite state Markov chains and algebras associated to etale groupoids. For further information contact Robert.D.Gray@uea.ac.uk. The visit is supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Niels Bonneux

Mr Niels Bonneux (Katholieke Universiteit Leuven, Belgium) will visit the University of Kent from 12 to 16 August 2019. Niels works on the classification and properties of exceptional orthogonal polynomials. He will give a seminar on the *Coefficients of Wronskian Appell Polynomials*. For further information email Clare Dunning (tcd@kent.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant

Visit of Dr Marzieh Forough

Dr Marzieh Forough (Czech Academy of Sciences) will visit the University of Glasgow from 1 to 14 July and from 18 to 23 August 2019. Dr Forough studies operator algebras with an emphasis on the structure of traces, and related tracial approximations. Talks for both visits will be at the University of Glasgow; for details contact stuart.white@glasgow.ac.uk. Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Alan Koch

Professor Alan Koch (Agnes Scott College, Georgia, USA) will visit Keele University from 8 to 18 July 2019. His recent research has concerned Hopf algebras, Hopf-Galois theory, and skew left braces. He will give a talk at the University of Exeter (11–12 July). For further details contact P.J.Truman@Keele.ac.uk. The visit is supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Victor Snaith

Professor Victor Snaith (Bristol) will visit the University of Sheffield from 16 to 20 September 2019. His research interests are in Algebraic Topology, Number Theory and Representation Theory. He will give a series of lectures on *Monomial Resolutions and Applications to Number Theory*. For further details contact J.Manoharmayum@sheffield.ac.uk. Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Reinhard Richter

Dr Reinhard Richter (Experimentalphysik 5, University of Bayreuth, Germany) will visit the University of Surrey from 2 to 7 July 2019. He is an experimentalist who focusses on experiments using ferrofluids. He will give a colloquium talk on *Pattern Formations in Ferrofluids*. For further details contact m.turner@surrey.ac.uk. Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Doug Stinson

Professor Doug Stinson (University of Waterloo, Canada) will visit Birkbeck, University of London from 25 August to 7 September 2019. His research interests include cryptography and computer security, combinatorics and coding theory, and applications of discrete mathematics in computer science. For further details contact Maura Paterson (m.paterson@bbk.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Jim Agler

Professor Jim Agler (University of California at San Diego, USA) will visit Newcastle University from 1 to 23 September 2019. His research interests include operator theory and the theory of analytic functions of several complex variables. For further information email Zinaida Lykova (Zinaida.Lykova@newcastle.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Pablo Shmerkin

Pablo Shmerkin (Torcuato di Tella, Buenos Aires) will visit the UK in autumn 2019. His research interests span fractal geometry, ergodic theory and harmonic analysis. During his visit, he will give lectures at the Universities of St Andrews, Edinburgh and Warwick. For further details contact Jonathan Fraser (jmf32@st-andrews.ac.uk). Supported by an LMS Scheme 2 grant.

Visit of John Oprea

Professor (Emeritus) John Oprea (Cleveland State University) will visit Queen Mary, University of London from 2 to 16 September 2019. His research interests are at the interface of algebraic topology and geometry, lately focussing on the notion of topological complexity in robotics. For further information contact Michael Farber (m.farber@qmul.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Igor Velčić

Dr Igor Velčić (University of Zagreb) will visit the University of Bath from 16 to 30 September 2019. Igor's research interests are in partial differential equations, calculus of variations, spectral theory, and their applications to multi-scale analysis and wave propagation. For further information email Kirill Cherednichenko (k.cherednichenko@bath.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

LMS Council Diary: A Personal View

Just over two months after the Strategic Retreat, Council met on Friday 12 April 2019 at De Morgan House. Naturally, some of the issues discussed at the Retreat such as the Standing Orders Review and a possible Atiyah memorial were discussed further. We had a full programme awaiting us.

The President started with an update on her activities since the last Council meeting. Most notably, she reported that the Bond Review response document had been updated following comments from members of Council during the retreat, and had been sent to the Chairs of the two CMS Committees for discussion at their next meetings. She also reported that she attended a live streaming of the announcement of the winner of the Abel prize 2019, Karen Uhlenbeck, an event held in partnership with the Norwegian Academy of Sciences. This was the first such event; disappointingly, attendance was rather low.

We then discussed Michael Atiyah tributes. Sufficient funding had been obtained to hold a Michael Atiyah memorial conference at the Isaac Newton Institute in September 2020; Nigel Hitchin had agreed to chair the scientific organising committee. However, concerns were voiced that the proposed venue might be not large enough, given that the event promised to be very popular. It was agreed to investigate whether recording or live streaming of the event would be feasible. We then discussed other potential tributes, which could be organised solely by the LMS. Council decided to focus on a potential research retreat, and a scholarship to support mathematicians from the Middle East to visit the UK, as appropriate memorials to Michael Atiyah.

As mentioned above, the Standing Orders Review was also discussed. Only small and final changes had to be made, and it was decided to send the tracked document for the first, informal consultation to the Privy Council, which is required before the document can be voted on by the membership. To give more time for preparation for the membership vote, we agreed to move the AGM to 29 November 2019.

We heard updates from the Prizes Committee and the Women in Mathematics Committee, and discussed financial matters including the Half-Year Review 2018/19 and the Indicative Operational Plans for 2019/20. Committee Membership was also discussed. It was noted that there are about 250 roles, and vacancies are regularly coming up. Hence we wondered whether it was advisable to advertise in some form in the *Newsletter*, which might be difficult to be timely as the *Newsletter* is only published bimonthly. The General Secretary reported that David Singerman is retiring from the *Newsletter* Board, and that the Editor-in-Chief would be stepping down at the end of his term of office. Council thanked the Editor-in-Chief for the tremendous work put into revising the *Newsletter* and for the success of the new format.

We received the Annual Reports from the Publications Committee and the Research Grants Committee, and received an update from the Society Lectures and Meetings Committee. The Chair of the Research Grants Committee reported that there had been an increase in grant applications since 2014, and he provided a list of common reasons for rejection of applications, noting that the failure of inviting a sufficient number of female mathematicians was especially prevalent. We finished with a very interesting discussion on the draft of the National Benchmarking Survey presented by the Chair of the Women in Mathematics Committee.

Brita Nucinkis

Philippa Fawcett Collection

The Philippa Fawcett Collection is a wide-ranging library of some 200 books written by and about women who studied or worked in mathematical subjects in the nineteenth and first part of the twentieth century, or earlier. A copy of the current catalogue can be found on the Society's website: lms.ac.uk/library/special-collections#fawcett.

The Collection was donated to the London Mathematical Society by one of its members, A.E.L. Davis, in the hope that the books will be a useful resource to scholars of the history of women in mathematics as well as an inspiration to female mathematicians of the future. Dr Davis named the Collection in honour of the first woman to come top in the finals examination, in 1890, of the Mathematical Tripos at the University of Cambridge. In those days, women could not be ranked alongside men, so instead, Fawcett was described as 'above the Senior Wrangler'.

Members are welcome to access the Collection during weekdays from 9.00 am–5.00 pm when visiting the Verblunsky Members' Room at De Morgan House (57–58 Russell Square, London WC1B).

Celebrating New Appointments (Scheme 9)

The Early Careers Research Committee would like to extend its warmest congratulations to the mathematicians below, who were awarded a Scheme 9 'Celebrating New Appointments' grant in the 2018/19 financial year, in celebration of their first appointments at a UK institution. The Early Careers Research Committee awarded £11,544 in the 2018/19 financial year to support the above one-day meetings. 'Celebrating New Appointments' grants are designed to provide partial support for meetings held in the United Kingdom, the Isle of Man and the Channel Islands, to celebrate the new appointment of a lecturer at a UK university. For more information, please visit lms.ac.uk/grants/celebrating-new-appointments-scheme-9.

Grant Holder/Institution	**Meeting**	**Date of Meeting**
Daniel Bearup (Kent)	Applications of Mathematics in Ecology	19 September 2019
Chris Bowman (Kent)	Lie Theory Days	11 September 2019
Andrea Brini (Birmingham)	Geometry and Physics of Mirror Symmetry	2 April 2019
Jeremiah Buckley (King's College London)	Analysis, Probability, Number Theory and Their Interaction	5 April 2019
Asma Hassannezhad (Bristol)	Spectral Theory and Geometry	14 May 2019
Tyler Kelly (Birmingham)	Mirrors in the Midlands	1 April 2019
Manuele Leonelli (Glasgow)	1st UK Workshop on Probabilistic Reasoning using CEGs	7 February 2019
Jing Li (Leeds)	Stochastic Models in Risk Analysis and Queuing	22 February 2019
Anna Maltsev (Queen Mary University of London)	Gaussian Fields: Geometry and Applications	13 June 2019
Stephen Muirhead (Queen Mary University of London)	Gaussian Fields: Geometry and Applications	14 June 2019
Neil Saunders (Greenwich)	A Day of Representation Theory in Greenwich	16 April 2019
Nicholas Simm (Sussex)	Random Matrices and Applications	16 September 2019
Raphael Stuhlmeier (Plymouth)	Water Waves — Mathematical Theory and Applications	5 September 2019
Jimmy Tseng (Exeter)	Dynamics, Number Theory and Geometry @ Exeter	5 April 2019
Daniele Valeri (Glasgow)	Integrability, Algebra and Geometry	13 December 2019
Minmin Wang (Sussex)	Branching Processes and Their Applications	17 September 2019
Sven-Ake Wegner (Teesside)	Analysis of and Analysis on Networks	22 March 2019
Thomas Woolley (Cardiff)	New Frontiers in Pattern Formation	3 December 2018
Jing Yao (Heriot-Watt)	Dependence Modelling and its Applications in Insurance and Finance	10 October 2019
Anna Zhigun (Queen's University Belfast)	PDE Models for Cancer Invasion	6 December 2019

Report: LMS Education Day 2019

LMS Education Day attendees

The 2019 LMS Education Day, organised by the LMS Education Committee, was held at De Morgan House on 13 May 2019. Colleagues from across the UK gathered to share ideas on how we can equip our mathematical sciences graduates, both at undergraduate and postgraduate level, with the skills above and beyond mathematical expertise that they need to get good jobs and become effective mathematicians in the workplace.

The day commenced with a summary from Mary McAlinden (University of Greenwich) of where we are with the subject-level Teaching Excellence Framework, which is now in a second pilot phase.

The second session of the day was on how we might teach ethics within the undergraduate mathematics curriculum, led by Maurice Chiodo (University of Cambridge). Maurice made a convincing case that ethics should be an integral part of our mathematical training. We train our graduates to use tools of great power in society but rarely discuss the ethics of using these tools, unlike other disciplines such as law, medicine and engineering where ethical use of skills is a mandated part of the curriculum. In discussion it was noted that the IMA's professional Chartered Mathematician registration scheme requires people to sign up to agree to use mathematics ethically. This is a positive step but have their studies prepared them to do this?

After lunch we focused on employability more generally, and how we might support our undergraduates and postgraduates in developing a range of useful skills needed for employment.

Michael Melgaard from the University of Sussex, which has the highest rate of mathematics graduate employability in the UK, spoke about what Sussex does to achieve this. Actions include starting early with first year students to get them thinking about careers, a compulsory employability-focused module in second year and support for careers including an embedded careers officer within the School. In further discussion, a number of others shared their experiences of embedding transferrable skills in the curriculum as part of preparing students for employment.

Employability skills for PhD students, whether they plan to continue in academia or to go into industry, is an area that has not often been formally addressed outside CDTs. Sam Cohen (University of Oxford) described the Fridays@4 seminar series for mathematics PhD students. Topics have included careers outside academia, academic writing, mathematics outreach and how to make the most of conferences, as well as mathematical seminars including a number of cross-disciplinary talks. This model is transferable to other Departments.

Jane White (University of Bath) led a session on how we can support and encourage our undergraduates to consider teaching as a career. Coming out from this the LMS Education Committee plans to produce a good practice guide on how departments can encourage students to consider teaching as a career.

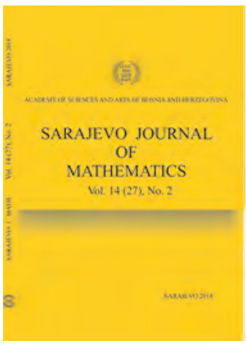
The LMS Education Day provides an opportunity to meet colleagues from across the UK to share practice in aspects of curriculum development and gain a greater understanding of the wider contexts in which our courses sit.

Cathy Hobbs
LMS Vice-President

Report: Visit to Bosnia and Herzegovina

Many people still remember the terrible civil war that struck the former Yugoslavia in 1991, which was particularly harsh on Bosnia and Herzegovina (BiH). The Bosnian war, part of the Yugoslav Wars, began in 1992 and ended in 1995, and local mathematicians have made great efforts to restore mathematics to the level it had been at previous to the war. This has been made harder by the fact that many known mathematicians and young talent left before or during the war. However, the University of Sarajevo continued to teach even during the war and Mathematics was kept alive. It was later joined by private or politically alternative universities. Namely, the peace agreement of 1995 divided the country into three rather independent pieces, so each one has its own university. The situation does not help research in mathematics, which is always enhanced by collaboration — while collaborating with scientists from other parts of BiH is very complicated for the local scientists. This is why I think that BiH mathematicians living outside of BiH and other mathematicians internationally have a large role to play in assuring a healthy state of mathematics there. Working in the international, including the UK, environment is a privilege that is ours to share.

An instance of mathematical life that was able to reinstate itself after the war is the *Sarajevo Journal of Mathematics*, former Radovi Matematiki, edited by the BiH Academy of Sciences and Arts (ANUBiH). It is an international open access journal aiming to publish good papers in all areas of mathematics. I have been an editor since 2005 and have worked towards raising the scientific and organisational level. The journal suffered a large crisis in 2017 and was under the threat of stopping its production. The crisis left it in need of a main co-editor who could handle the technical requirements of the production and who could popularise the journal to international audiences. I was honoured to be nominated and have since been working to achieve the goals. The LMS grant I obtained for this collaboration allowed to make regular visits (it financed six visits) and collaborate with the other main co-editor Academician Mirjana Vuković. Together, we produced three issues of the journal. I have now stepped down from the role to allow the local forces to continue and further develop the journal.

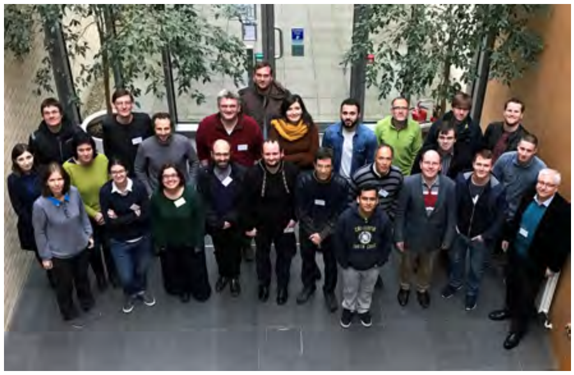


During the visits I also participated in two conferences: *Modern Algebra and Analysis*, in September 2018, organised by ANUBiH and *Bosanski Matematički Skup* in June 2019 organised by various universities in BiH. Following its commitment to help the members balance family and work life, LMS has allowed me to use a part of the funding to help childcare expenses for my 12 year old daughter whom I took to some of these trips.

This visit was supported by an LMS Scheme 5 grant.

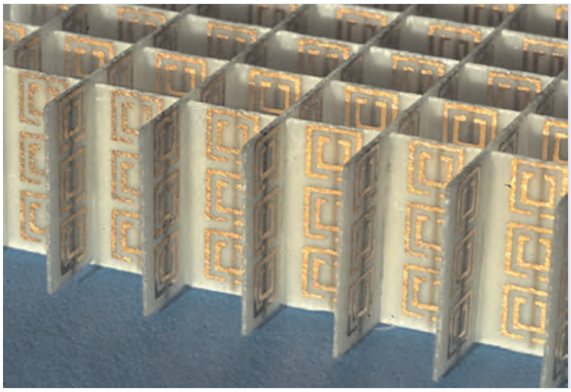
Mirna Džamonja
University of East Anglia

Report: Operators, Operator Families, and Asymptotics II



Conference attendees

The research conference *Operators, Operator Families, and Asymptotics II* took place from 14–17 January 2019 at the Department of Mathematical Sciences, University of Bath. The conference, which was the second in a series taking place every three years, presented an overview of the state of the art in a rapidly developing area of analysis concerned with application of the techniques of operator theory to the asymptotic study of parameter-dependent differential equations and boundary-value problems. The subject is currently maturing for new breakthroughs in the applications of mathematics to the real-world technologies that depend on understanding the behaviour of solutions to parameter-dependent boundary-value problems. One notable example of this is found in the development of techniques for manufacturing composite materials exhibiting negative refraction ('metamaterials'): analysis of operators and their families offers a cost-effective way to explore possible ways to obtain such composites, and vice versa, the practical needs provide a stimulus to further development of this classical area of analysis.



Example of a split-ring resonator metamaterial

The conference was attended by 30 specialists in the areas of analysis and its applications where the asymptotic behaviour of an operator family presents a mathematician with an exciting analytical challenge and a physicist with a new tool to address problems at the frontline of materials research. The talks ranged from the analysis of quantum graphs with singular potentials to function analytic techniques in scattering theory and the study of resonance phenomena in inhomogeneous media. Further details about the conference can be found at tinyurl.com/OOFA19-Info.

The conference was supported by an LMS Conference Grant, Newton Fund (Royal Society), EPSRC and Bath Institute for Mathematical Innovation.

Kirill Cherednichenko
University of Bath

Report: Hirst Lecture and Society Meeting

Jeremy Gray (left) and June Barrow-Green

The Hirst Prize and Lectureship for the History of Mathematics is awarded jointly by the LMS and the British Society for the History of Mathematics, for contributions to the study of the history of mathematics. In 2018 the prize was awarded to Professor Jeremy Gray, of the Open University, "for his research and books on the history of mathematics, especially differential equations and geometry in and around the nineteenth century".

Jeremy's Hirst Lecture, formed the core of a well-attended Society Meeting on 21 March 2019 at De Morgan House. An opening lecture, by Professor June Barrow-Green, completed the programme. The two lectures complemented each other well, both focusing on American mathematicians in the first half of the twentieth century.

June Barrow-Green spoke first, on *George Birkhoff: "The Poincaré of America"*. Her title, which she took from a quote by Nikolai Krylov in 1924, makes clear the strong connection between Birkhoff, and the legacy of Poincaré. However, one of June's points was that the legacy was intellectual rather than social: Birkhoff was among the first generation of leading American mathematicians to have made his career entirely in the United States. Unlike many of his predecessors, he did not study in Europe at either undergraduate or postgraduate level. He gained an international reputation, though, in 1912, a few months after Poincaré's death, by publishing a proof of Poincaré's "last geometric theorem" (now generally known as the Poincaré-Birkhoff theorem). June started her talk by telling us something of the publication history of the theorem, which Poincaré had succeeded in proving only in some special cases but was anxious to publish before he died. Having proved the theorem more generally, Birkhoff continued working on topics connected with Poincaré as he went on to develop dynamical systems theory, becoming one of the leaders of American mathematics in the first half of the twentieth century.

Following a tea break, Jeremy Gray delivered his Hirst Lecture, on *Jesse Douglas, Minimal Surfaces, and the first Fields Medal*. Jeremy chose to talk about the outcomes of his collaboration with Mario Micallef (Warwick) in studying Jesse Douglas, one of the first two Fields Medallists in 1936. Despite the high profile of Douglas' work on minimal surfaces — he was the first person to solve the Plateau problem for discs spanning an arbitrary contour, and to generalise the problem successfully to surfaces of arbitrary topological type — there are large gaps in our knowledge of his life. Like Birkhoff, Douglas was educated in the USA, but he toured Europe — particularly Paris and Göttingen — in the late 1920s. He had announced that he had a solution to the Plateau problem as early as 1926, and talked about it several times during his European tour, but it seems that it was not until 1931, following the discovery of his “A functional” that he finally sorted out all the details. Having outlined the nature of Douglas’ proof, and pointed out how distinct it was from that independently discovered by Tibor Radó, Jeremy discussed the evidence — or lack of it — for Douglas’ reception by the mathematical community, his lack of a secure institutional position despite his clear ability, and the gaps in his biography.

The meeting concluded with a wine reception, providing ample opportunity for chat and further discussion of issues raised by the lectures.

Isobel Falconer
University of St Andrews

Report: Christopher Zeeman 2018 Medal Winner Dr Hannah Fry

LMS President, Caroline Series, Hannah Fry, IMA President Alistair Fitt

The Christopher Zeeman Lecture took place at The Royal Society on 5 March 2019 at which Hannah Fry was presented with the IMA-LMS Christopher Zeeman Medal. This report is based on her lecture *Hello World*.

Hannah is the most brilliant of communicators of mathematics. Coupled with her clear, engaging exposition, Hannah has that very rare quality of being able to ‘take the mathematics’ to her audience — this was the overarching theme in her lecture. She explained that because at first sight mathematics doesn’t have the ‘awe and wonder’ that other subjects have, such as physics with the Higgs-Boson, black holes, etc we can’t expect the public etc to be interested and engaged in maths so instead she must ‘take the mathematics to her audience’. Hannah said that we should do the same — as mathematicians we should find out ‘where our audience is’ and ‘take the mathematics to them’ — they are likely to be more interested, eager to engage and learn more, and to have a better appreciation and understanding of what mathematics is, how it relates to and benefits them, and what it can do for us all.

Hannah began her lecture by telling us how previous winners had inspired her as she was growing up and becoming ever more interested in mathematics, and computers.

The lecture was full of fascinating facts and insights across a very wide range of topics — from pigeon ‘outliers’, through riots, cows wearing pedometers, Bach cantatas, dementia, to pink sheep and nuns, and much else besides, concluding with radiologists not seeing a picture of a gorilla on a CT scan.

So, what relates this seemingly disconnected set of topics that Hannah spoke about? In short — humans and machines: their relationship, how we can make the best of our ability to give insight, weigh up the evidence, take a balanced view to make a judgement; complemented by the ability of machines and their algorithms to process vast amounts of data, with their sheer power and speed.

Through a series of examples Hannah demonstrated how the creation of such strong relationships, and an understanding of the strengths and weaknesses of each, can work to good effect and that without this there can be disastrous consequences.

Hannah explained that care is needed when taking many decisions, like those that occur in medicine, because some decisions are a matter of life and death, or at least may have life-changing consequences. She summarised this very neatly as ‘sensitivity’ versus ‘specifics’, each represented by a ‘dial’ that can be ‘turned’, but where the dials cannot be operated independently. Wherever one sets the dial on the ‘sensitivity’ scale will have an impact on the ‘specificity’, and vice versa, and that is where the human element comes in. She gave the example of breast cancer screening — it needs to be ‘sensitive’ enough to pick up on the abnormalities present in all the breasts that have tumours, without missing any pixels in the image with tumorous tissue and reporting that these are ‘clear’. On the other hand, it needs to be ‘specific’ enough not to flag perfectly normal breast tissue as suspicious. This can be compared to hypothesis testing in statistics and the concept of 'false positives' (Type I errors) and 'false negatives' (Type II errors). A false positive occurs when the algorithm incorrectly identifies a healthy woman as having breast cancer — it is too sensitive; while a false negative occurs when the algorithm incorrectly identifies a woman with tumours as being healthy — it is not specific enough.

On the lighter side, Hannah talked about image recognition and machine learning algorithms — they can recognise objects in pictures and understand words as we speak them and translate from one language to another, i.e. from images to text.

You can find out much more about these examples, and many more, from Hannah's truly excellent book, *Hello World: How to Be a Human in the Age of the Machine* (Penguin, 2018).

Professor Paul Glaister
Department of Mathematics, and Statistics
University of Reading

Records of Proceedings at LMS Meetings
Hirst Lecture and Ordinary Meeting: 21 March 2019

The meeting was held at De Morgan House. Over 40 members and guests were present for all or part of the meeting. The meeting began at 2.30 pm with the President, Professor Caroline Series, FRS, in the Chair.

There were 43 members elected to Membership at this Society Meeting, as follows:

11 Associate Members: Mrs Noura Alharthi, Mr Ahmad Almutlg, Mr Adrian Bell, Mr Benjamin Eylott, Dr Tamas Gorbe, Mr Hassan Izanloo, Mr Barry Lawson, Dr Abdulfatai Momoh, Dr Eng-Jon Ong, Mr Allen Paul and Mr Ruairidh Percival.

One Associate Teacher Training Scholar Member: Mrs Isadora Farley.

26 Ordinary Members: Dr Robert Max Bauer, Professor Rachel Bearon, Professor Michael Bonsall, Dr Vincent Caudrelier, Professor Emeritus Manos Digalakis, Dr Joanna Fawcett, Dr Felix Fischer, Professor Michael Giles, Dr Jurriaan Gillissen, Dr Mainul Haque, Dr Catalin Mugur Iorga, Professor Delaram Kahrobaei, Dr Katerina Kaouri, Professor Mark Verus Lawson, Mrs Jo Lees, Mr Michael Edward Alexander Lyons, Dr Thomas Madsen, Mr Amar Moustafa, Ms Vesna Perisic, Dr Dhruv Ranganathan, Dr Mikhail Soloveitchik, Dr Bjorn Stinner, Mr Harkaran Singh Taggar, Mr Hosea Yakubu, Dr Ewelina Zatorska and Dr Anna Zhigun.

Five Reciprocity Members: Dr Olukayode Adebimpe, Mr Elzein, Dr Ivan Kryven, Dr Mohammad Sajid and Dr Anandhi Santhosh.

Six members signed the Member's Book and were admitted to the Society.

Professor Caroline Series asked all present whether the Records of Proceedings for previous Society Meetings – Joint Meeting with the Fisher Trust, RSS, Galton Institute and Genetics Society on 9 October 2018, the Annual General Meeting on 9 November 2018 and the South West & South Wales Regional Meeting on 17 December 2018 – were a true and accurate record of what happened, and signed the Records of Proceedings as accurate.

Professor Series introduced Professor June Barrow-Green of The Open University, who gave the opening lecture titled *George Birkhoff: "The Poincaré of America"*.

After tea, Professor Series introduced Professor Jeremy Gray of The Open University and the University of Warwick, who then gave the Hirst Lecture titled *Jesse Douglas, Minimal Surfaces, and the first Fields Medal.*

Professor Series warmly thanked the speakers and the organisers of the meeting, the Society, Lectures and Meetings Committee, for a fascinating event and a delightful day.

The meeting closed at 5.15 pm, and was followed by a wine reception at De Morgan House, and a Society Dinner in The Blue Door Bistro, Montague on the Gardens Hotel.

Records of Proceedings at LMS Meetings
Mary Cartwright Lecture and Society Meeting: 5 April 2019

The meeting was held at the International Centre for Mathematical Sciences (ICMS), Edinburgh. Approximately 30 members and visitors were present for all or part of the meeting.
The meeting began at 3.30 pm with Vice-President Professor Catherine Hobbs in the Chair.
There were no elections to membership.
Three members signed the book and were admitted to the Society.
Professor Hobbs then handed over to the Chair of the Women in Mathematics Committee, Dr Eugenie Hunsicker, who gave an introduction about the mathematical life and achievements of Professor Dame Mary Cartwright, FRS.
Dr Hunsicker then introduced a lecture given by Professor Karima Khusnutdinova (Loughborough University) on *Integrable and Near-Integrable Models for Surface and Internal Waves in Stratified Shear Flows*.
After tea, Dr Hunsicker introduced the Mary Cartwright Lecture given by Professor Beatrice Pelloni (Heriot-Watt University) on *Nonlinear Transforms in the Study of Fluid Dynamics*.
Vice-President Professor Hobbs expressed the thanks of the Society to the Women in Mathematics Committee and the ICMS for organising and hosting a successful meeting.
Afterwards, a reception was held at the ICMS, followed by dinner at Blonde Restaurant, Edinburgh.

Records of Proceedings at LMS Meetings
Ordinary Society Meeting at the British Mathematical Colloquium: 10 April 2019

The meeting was held at the George Fox Building, the University of Lancaster, as part of the British Mathematical Colloquium 2019.
Over 50 members and guests were present for all or part of the meeting.
The meeting began at 4.40 pm, with the President, Professor Caroline Series, FRS, in the Chair.
There were no members elected to Membership at this Society Meeting.
Three members signed the Members' Book and were admitted to the Society by the President during the meeting, and in addition the President extended an invitation to those present to look through the Members' Book during the wine reception.
There were no Records of Proceedings confirmed at this meeting.
Professor John Greenlees, the Vice-President, introduced the lecture given by Professor Kathryn Hess (École Polytechnique Fédérale de Lausanne) on *Topological Adventures in Neuroscience*.
The Vice-President thanked Professor Hess for her talk, and the President further extended her warm thanks to Professor Hess and to the local organiser, Dr Jan Grabowski for organising and holding such an exemplary and interesting colloquium.
Dr Grabowski invited all those present to attend a wine reception held in the foyer of the George Fox Building, sponsored by Cambridge University Press. A Colloquium Dinner was held following the wine reception at Barker House Farm.

CORRECTIONS AND CLARIFICATIONS

In the May 2019 issue (482) of the *Newsletter*, the obituary for Lilian Grace Button, on page 39, should have recorded that she attended Christ's Hospital School, Hertford, Hertfordshire, from January 1934 to July 1942. She did not attend Charterhouse, nor was there a girls' school with that name.

Fifty Years of Thermodynamic Formalism

NATALIA JURGA

When we examine the interplay between mathematics and the sciences, we usually see that mathematics forms the basis on which scientific disciplines are built. However, here we explore the birth and evolution of an inverted case in which a mathematical theory was cultivated from the ideas of statistical physics.

Dynamical systems

Dynamical systems is an area of mathematics concerned with the study of the long term behaviour of points in a space X (called the phase space) under the iteration of a map $T : X \to X$, which we call a dynamical system[1]. It is a very active area of both pure and applied mathematics with applications to areas of pure mathematics such as number theory (see "The Gauss map") and geometry, and 'real world' applications to fields such as biology, economics and meteorology.

Some basic questions that a dynamicist might be interested in answering about a dynamical system $T : X \to X$ are given below.

- Are there any periodic points? A point $x \in X$ is called periodic if there exists $n \in \mathbb{N}$ such that $T^n(x) = x$, where $T^n = \underbrace{T \circ \cdots \circ T}_{n \text{ times}}$.
- Are there any invariant measures? A measure μ on X is called invariant if measure is preserved by the dynamics of T, that is $\mu(T^{-1}(A)) = \mu(A)$ for any measurable $A \subseteq X$.
- Are there any attracting sets (sets of points which attract nearby points) or repelling sets (sets which repel nearby points)? If yes, what does such a set look like — for instance what are its geometric properties?
- Does the dynamical system have sensitive dependence on initial conditions? This means that the orbits of points which are initially close eventually become very separated, where the orbit of a point x is its path $(x, T(x), T^2(x), \ldots)$ under the dynamics. This is a central characteristic of what is known as a chaotic dynamical system.

The Gauss map

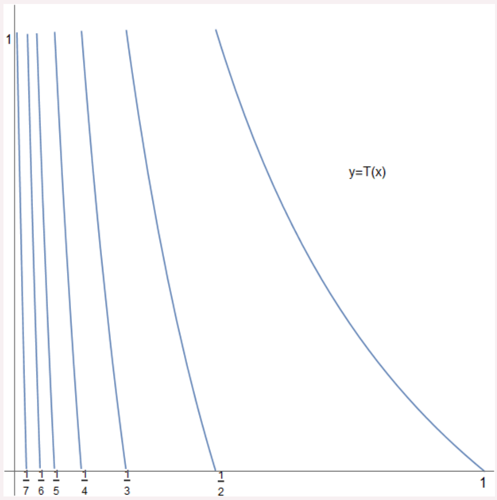
Let $x \in [0, 1]$ be an irrational number. Then there exists a unique sequence $\{a_n\}_{n\in\mathbb{N}}$, of natural numbers $a_n \in \mathbb{N}$, such that

$$x = \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}}$$

which is known as the continued fraction expansion of x. Define the Gauss map $T : [0, 1] \to [0, 1]$ given by

$$T(x) = \begin{cases} 0 & x = 0, \\ \frac{1}{x} \mod 1 & x > 0. \end{cases}$$

It is not difficult to show that $T^n(x) \in (\frac{1}{k+1}, \frac{1}{k}]$ if and only if $a_n = k$. This connection makes the Gauss map very useful for studying statistical properties of the continued fraction expansions of numbers.



[1] More precisely this is a discrete dynamical system where time evolves in discrete units, as opposed to a continuous dynamical system where time changes continuously.

The origins of the mathematical field of dynamical systems date back to the work of Henri Poincaré in the late nineteenth century. More recently, interest in dynamical systems grew significantly following the birth and subsequent popularisation of 'chaos theory' in the second half of the twentieth century, whose development was largely catalysed by the invention of the modern computer. In the 1960s the meteorologist and mathematician Edward Lorenz performed a rerun of a numerical weather model by using a rounded off initial input and famously came back to find a completely different weather prediction. This led to the phrase 'the butterly effect' which essentially describes the notion of sensitive dependence on initial conditions: a small difference in the initial input (such as the flap of a butterfly's wings in Brazil) can compound into a completely different evolution of the weather (such as a tornado in Texas weeks later).

Just as Lorenz's rerun of the weather simulation did not give away much information about the fate of the original simulation, when one studies a chaotic dynamical system at a 'microscopic level', for instance the behaviour of only a finite number of orbits, then this gives very limited information about the system as a whole. Therefore a different approach is required, one whose focus is the average or 'typical' behaviour of the system: this is the programme of ergodic theory.

Ergodic theory

It is impossible to tell the story of ergodic theory without first discussing statistical mechanics, whose pioneers are considered to be the fathers of ergodic theory.

Statistical mechanics, now a pillar of theoretical physics, originated with the aim of relating the microscopic behaviour of a system of single molecules, such as a box of gas molecules, to macroscopic observations of the system. For example, how could measurements of pressure or temperature be understood as averages over movements of individual particles within a system? In a way, the birth of statistical mechanics was essentially, quoting from [5],

> an acceptance of failure: the laws of classical mechanics, applicable for studying a fluid at the microscopic level, are completely useless at the macroscopic level due to the enormous number of interacting particles. So the only measurable quantities are averages and the movement of the particles must be thought of as random, and they must be examined statistically.

One of the first key influences of statistical mechanics on the mathematical field of dynamical systems was Boltzmann's *ergodic hypothesis* (1887). Ludwig Boltzmann was one of the founders of statistical mechanics (and is credited for coining the term 'ergodic' based on the Greek words for 'work' and 'path'), although his ideas met with a lot of criticism at the time. In order to formulate his ergodic hypothesis, suppose that we have a system that describes the behaviour of gas molecules in a box. By a configuration we mean the position and direction of motion of each gas molecule at a fixed point in time and by the phase space we mean the collection of all possible configurations. Boltzmann's ergodic hypothesis says that over long periods of time, the time spent by the system in a given region A of the phase space will be proportional to the 'size' of A.

Boltzmann's ergodic hypothesis eventually gave birth to the *Birkhoff ergodic theorem* (1931), now a cornerstone of the ergodic theory of dynamical systems. Although there is a version for invariant measures, we will state the version for ergodic probability measures, where a probability measure μ is said to be ergodic if, from the point of view of the measure μ, there are no subsets of the space X which are invariant under the dynamics, i.e. $T^{-1}A = A$ only if $\mu(A) = 0$ or $\mu(A) = 1$.

Theorem 1 (Birkhoff's ergodic theorem). *Let μ be a probability measure on the space X which is ergodic for $T : X \to X$. Then for every integrable function $f : X \to \mathbb{R}$,*

$$\frac{1}{n}\sum_{i=0}^{n-1} f(T^i x) \to \int f \, \mathrm{d}\mu$$

for μ almost every $x \in X$.

We call $\sum_{i=0}^{n-1} f(T^i x)$ a *Birkhoff sum*. Birkhoff's ergodic theorem says that along a *typical* orbit, the time average of f equals the space average of f. The purpose of the function f in the theorem is to measure some 'characteristic' of each point, and therefore is sometimes called an observable. To demonstrate the connection with Boltzmann's ergodic hypothesis, we could take $f : X \to \mathbb{R}$ to be the function

$$f(x) = \begin{cases} 0 & x \notin A \\ 1 & x \in A \end{cases}$$

so that f indicates whether a given point belongs to a measurable set $A \subseteq X$ or not. Then, considering each point x as playing the role of a configuration, X as the phase space of all possible configurations and A as some subset of the phase space, the theorem says that the proportion of time $T^n x$ spends in A is equal to $\mu(A)$ for a typical configuration x, which is precisely Boltzmann's ergodic hypothesis.

Thermodynamic formalism

Although historically ergodic theory was a branch of statistical mechanics and therefore both fields evolved more or less in parallel, much more pronounced application of ideas from statistical mechanics into the mathematical field of dynamical systems really set off in the 1970s, primarily due to Yakov Sinai, David Ruelle and Rufus Bowen.

From left to right: Yakov Sinai (1935–), David Ruelle (1935–), Rufus Bowen (1947–1978). (Courtesy of the Archives of the Mathematisches Forschungsinstitut Oberwolfach.)

Firstly, different mathematical notions of entropy and pressure were introduced and were shown to be related to one-another by an elegant formula known as the *variational principle*. Following this, a theory of *equilibrium measures* was developed which drew upon the notion of equilibrium from thermodynamics. These equilibrium measures were shown to have properties which were analogous to the distributions in statistical mechanics introduced by Boltzmann and studied extensively by Josiah Gibbs (who coined the term 'statistical mechanics'), and therefore became known as *Gibbs measures*. This entire theory was bound together by a beautiful generalisation of the famous Perron–Frobenius theorem (which describes the spectrum of positive matrices) to operators, which is now known as the *Ruelle–Perron–Frobenius theorem*.

This framework and all of the subsequent generalisations, applications and techniques became collectively known as *thermodynamic formalism*. Over the last few decades the injection of more refined techniques from different areas of mathematics such as operator theory combined with a growing number of applications has resulted in an enormous body of work that is rich in both tools and scope. Although the intuition of statistical physics certainly guided the early pioneers in establishing the foundations of thermodynamic formalism, it has now evolved into a self-contained mathematical theory whose ongoing development is no longer dependent on understanding the physical origins of the theory. Indeed, as Zinsmeister [5] describes:

> This situation is exactly symmetric to the one with which we are accustomed, where physicists, to the great surprise of mathematicians, use abstract mathematical results to model experimental situations. Here it is mathematicians who superimpose the thermodynamical model over their problem. What has happened is not just the "borrowing" of ideas or intuition, but the direct and unabashed application of a physical model.

So far we have described the evolution of ideas, focusing more on the historical origin of thermodynamic formalism rather than on a formal description of any of the mathematical tools that it comprises. This is partly because the significance of the field is not in the actual tools and devices that make up its framework but in its many applications: initially to dynamical systems and consequently to many other areas of mathematics due to the usefulness of the dynamical systems framework. For example, a central tool that is used in thermodynamic formalism is an operator called the *transfer operator* which describes the action of the dynamics on mass densities. The transfer operator is useful for deducing statistical properties of the dynamical system in question, which can then be used to yield interesting consequences in areas of mathematics which have some underlying dynamical nature.

For the remainder of the article we focus our attention on the application of ideas from thermodynamic formalism to one such unlikely subject: fractal geometry. In particular, we will discuss a formula which allows one to calculate the dimension of certain fractals, where the central mathematical device in the formula is a conceptual descendent of the statistical mechanics notion of the pressure of a system of particles.

Applications to fractal geometry

Fractals are geometric objects that appear in different areas of mathematics which have a very intricate structure when viewed at any scale and often feature some form of self-similarity. Well-known examples include the Julia sets (Figure 1) which describe a particular mathematically defined set of complex numbers and the Cantor sets (Figure 2), which have a straightforward recursive construction.

Figure 1: For a fixed $c \in \mathbb{C}$, the Julia set J_c is the boundary of the set $\{z \in \mathbb{C} : |f_c^n(z)| \not\to \infty\}$ where $f_c(z) := z^2 + c$. In this figure $c = -0.835 - 0.2321i$.

The word 'fractal' was coined by Benoit Mandelbrot from the Latin 'fractus' meaning 'broken' to describe these objects which do not fit into the setting of classical Euclidean geometry. Indeed their unusual geometric properties can be quantified: the dimension of a fractal (see "What is the dimension of a set?") is often different from the dimension of the ambient space, and usually is not an integer.

Fractals are closely related to dynamical systems since they often appear as subsets of the phase space which are invariant under the dynamics. For example many expanding dynamical systems (ones where nearby points get increasingly separated from each other under iteration by T) have an invariant repelling set which captures the interesting part of the dynamics, and this set is often a fractal. For example, consider the map obtained from the usual Gauss map by removing some branches. Then there exists a fractal subset of $(0, 1)$ which is invariant under the modified Gauss map. Moreover, from the point of view of number theory this fractal set corresponds to the irrationals whose continued fraction expansion only contain digits from a restricted

What is the dimension of a set?

There are many different mathematical definitions for dimension but they all formalise the everyday notion of dimension. For example, dimension can be thought of as describing how efficiently one can cover a given object by sets of some fixed diameter; a line of unit length can be covered by roughly $(\frac{1}{r})^1$ lines of side r, a square of side 1 can be covered by roughly $(\frac{1}{r})^2$ squares of side r and so on. This observation leads to the notion of the *box dimension* of an object F which, roughly speaking, is defined as the smallest exponent d for which F can be covered by roughly $(\frac{1}{r})^d$ sets of diameter r, for any scale r.

subset of $\mathbb{N}$ (those numbers corresponding to the branches that weren't removed).

Figure 2: The construction of the middle third Cantor set. A different Cantor set can be obtained by changing the number and sizes of intervals that get removed.

Thermodynamic formalism underpinned much of the progress in fractal geometry over the last fifty years. One of the first important contributions of thermodynamic formalism to fractal geometry was a formula due to Bowen that related the dimension of sets which were left invariant under certain dynamical systems with the root of a 'pressure equation', where the origins of the mathematical definition of the pressure, denoted below by P, lie in the statistical mechanics definition of the pressure of a system of a large number of particles. A special case of Bowen's formula is stated below.

Theorem 2 (Bowen's formula). *Let $\{I_n\}_{n=1}^N$ be a sequence of disjoint closed subintervals of $[0, 1]$ and $T : \bigcup_{n=1}^N I_n \to [0, 1]$ be a piecewise C^2 expanding full branched map of the unit interval. Then there is a set $\Lambda \subset \bigcup_{n=1}^N I_n$ which is invariant under T and whose dimension $s = \dim \Lambda$ is given by the unique solution s to*

$$P(-s \log |T'|) := \lim_{n\to\infty} \frac{1}{n} \log \sum_{T^n x = x} \frac{1}{|(T^n)'(x)|^s} = 0.$$

To demonstrate a simple application of this theorem, consider a Cantor set construction where we begin by keeping only k disjoint intervals $\{I_i\}_{i=1}^k$, each of length r_i whose left end-point will be denoted by x_i (so that $x_1 = 0$ and $x_k = 1 - r_k$). Let Λ be the Cantor set which is obtained by the usual construction, see Figure 3. Define a dynamical system $T : \bigcup_{i=1}^k I_i \to [0,1]$ by $T(x) = \frac{1}{r_i}x - \frac{1}{r_i}x_i$ on the ith interval, as in Figure 3. Then T is a piecewise C^2 expanding full branched map and Λ is invariant under T.

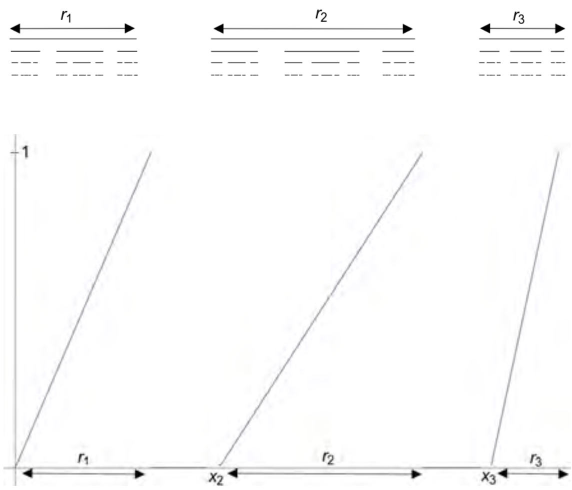


Figure 3: The top picture shows the first few steps in the construction of Λ. The second picture is the dynamical system which leaves Λ invariant.

By linearity of the branches,

$$P(-s\log|T'|) = \lim_{n\to\infty}\frac{1}{n}\log\sum_{i_1,\dots,i_n\in\{1,\dots,k\}}(r_{i_1}\cdots r_{i_n})^s = \log\sum_{i=1}^k r_i^s,$$

thus by applying Bowen's formula it follows that $\dim\Lambda$ is the unique solution s to

$$\sum_{i=1}^k r_i^s = 1.$$

In fact, a version of Bowen's formula is true in much greater generality. In general, the map T can be replaced by any sufficiently smooth conformal map of a Riemannian manifold which is expanding and 'mixing' on its repeller, where conformal means that the rate of expansion in all directions is the same (which is of course immediate when considering maps on the unit interval). In fact, Bowen's original result was stated as a formula for the dimension of Julia sets (Figure 1). On the other hand, modern research predominantly deals with non-conformal settings where points are being expanded by different amounts in different directions by the map T. This yields a T-invariant set which is more 'natural looking' but more difficult to study since it is made up of 'distorted' copies of itself and unfortunately techniques from the conformal setting do not transfer across. However, by letting sub-additive sequences of functions (which arise naturally in the non-conformal setting) replace Birkhoff sums (which arise naturally in the conformal setting) one can yield analogues of many theorems of classical thermodynamic formalism. This has led to substantial progress in the non-conformal setting and has formed a branch of thermodynamic formalism known as 'sub-additive thermodynamic formalism' which is enjoying ongoing development and offers exciting directions for future research.

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Mathematician in a Minefield

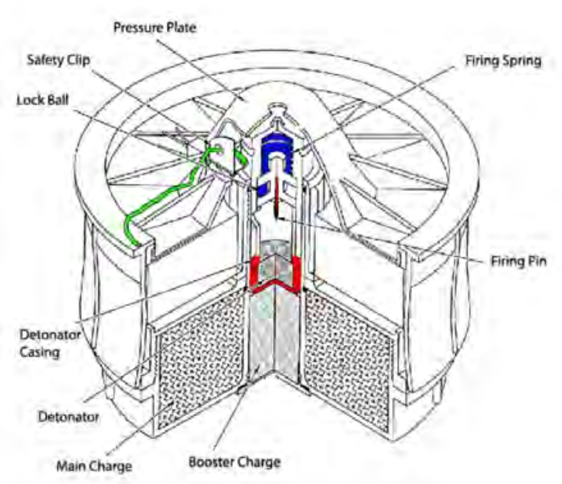
BILL LIONHEART

Detecting minimal-metal land mines is a big problem in civilian mine clearance. A lot of time is taken up with false alarms from small fragments of metal. Can mathematics help in the design of better metal detectors?

April 2017: It is 40 degrees centigrade and the sun beats down on a field in Battambang district in Cambodia. After a safety briefing that included the different coloured markers for areas declared safe and those that needed a second check, and where the helicopter landing point was in case of a need for medical evacuation, we donned heavy vests and helmets with visors to protect us from blast. We set off behind our guide. We knew the path had been cleared but I have never been so suspicious of a piece of ground, finding myself treading in footprints where I could. We got to the edge of the cleared area where a deminer working for the Cambodian Mine Action Committee (CMAC) was at work. As well as a metal detector, he had a thin pointed rod to probe the ground and a pair of gardener's secateurs to clear vegetation. His machine had registered some metal and he knelt on the cleared ground carefully probing to check if he could find the plastic casing of an anti-personnel land mine. Once he knew there was no mine, he still had to find the metal to be sure it was clear and after 15 minutes of careful work he shows us what he found. Three small fragments of rusty metal, that could be remnants of shrapnel or just a discarded tin can. And then on to clear the next 30cm of his designated lane, and so on shift after shift.

How were we drawn into the minefield?

Millions of anti-personnel land mines contaminate land in rural locations. They persist as a danger long after armed conflict has ceased, often used in an unsystematic and undocumented way, they are inexpensive and easy to deploy. Typically they are constructed using a minimum of metal components to make them difficult to detect with metal detectors. Often only a small firing pin and percussion cap is metallic (see the figure). Post conflict, the process of civilian demining is slow and laborious, and uses a metal detector to detect all the metal objects to verify there is no mine. In 2007 famous footballer Sir Bobby Charlton visited a mine field near Battambang in Cambodia with the mine clearance charity MAG. He was appalled to see children with limbs missing as a result of injury from mines, and he was determined to help find a better way to clear land mines. He approached a group of physicists, engineers and mathematicians at the University of Manchester, together with MAG and the security company Rapiscan Systems, with whom we had worked on security screening metal detectors.



Metalic content in a plastic anti-personnel mine. The minimum, additional and removable metallic content is shown in red, blue, and green, respectively. (Adapted from Collaborative ORDnance Data Repository cord.gichd.org)

Many of us had thought about mine clearance before. There are some remarkably ingenious methods deployed, some of them to detect the vapour from explosives. For example, a species of wild flower that is genetically engineered to grow flowers of a different colour when near explosives. We were told by MAG that many of these amazing ideas turned out not to be usable in the field. At that point we decided, if we kept in close contact with demining agencies, that we might come up with pragmatic improvements in mine clearance that were really useful. Metal detectors had been in use for mine clearance since World War II, and while the electronics is now more sophisticated, the basic principle is still the same. A low frequency alternating magnetic field is changed by the presence of a conducting object, and that change is detected. Another approach to detecting things underground is to use Ground Penetrating Radar (GPR) in which microwaves are directed into the ground and the reflections off buried objects detected. These are widely used in civil engineering and archaeology, typically a system that looks a bit like a lawn mower. Simple hand held systems were beginning to be available that combined GPR and metal detectors, the idea being to reduce false positives and speed up demining.

A CMAC deminer using a metal detector

Inverse boundary value problems.

My work is on inverse problems, and when asked by lay people I generally say that I make three dimensional images of the inside of objects from measurements outside. The applications of this I have worked on range from medical imaging to security screening and non-destructive testing. The mathematics typically involves finding an unknown coefficient in a (system of) partial differential equations from overspecified boundary data. An archetypal example is Calderón's inverse boundary value problem. Let Ω be a bounded domain in two or three dimensional Euclidean space and $\gamma : \Omega \to \mathbb{C}$ be an essentially bounded function with real part bounded from below. The (weak) boundary value problem, to find u when

$$\begin{aligned} \nabla(\gamma \nabla u) &= 0 \\ u|_{\partial\Omega} &= f, \end{aligned}$$

is well posed when γ is known. Let $\Lambda_\gamma(f) = \gamma \nabla u \cdot n$ where n is the outward unit normal to $\partial\Omega$; this is called the Dirichlet-to-Neumann map. If γ is a complex electrical conductivity, then f is the voltage at the boundary and $\Lambda_\gamma(f)$ is the current density at the boundary. By applying voltages and measuring currents (or vice versa) at the boundary, we get to know an approximation to Λ_γ. Calderón's problem is to recover γ from Λ_γ. This is known to have a unique solution in dimension two, and, with additional regularity assumptions on γ, in higher dimensions. In applications, the problem arises as electrical impedance tomography in geophysics, medical imaging and process monitoring [1]. The model behind this application is an approximation to Maxwell's equations in which magnetic fields are ignored.

One practical example I had worked on with my engineering colleague Tony Peyton was monitoring the flow of molten steel in continuous casting. It was this concept, working with Rapiscan Systems, that we also applied to improve metal detectors for airports. Rather than just beeping if metal was detected, we could provide an image of the metal. We made a prototype system and tested it at Manchester Airport. So when Sir Bobby drew us into working on mine detection, the idea we thought we were best able to help with was improving metal detectors so that they could locate and identify multiple objects.

Fish and polarization tensors

In land mine detection, rather than having a continuum of variable conductivity, we typically have a few isolated highly conducting objects. An analogous problem with the simple scalar problem of electrical impedance tomography was solved about ten million years ago by weakly electric fish. These fish generate small electrical voltages, unlike strongly electric fish that generate voltages high enough to stun prey, they use the electric source, and an ability to sense voltage on the surface of their body, to locate and identify prey. A common mathematical model for this (see [5]) is to consider the 'prey' or other target object to be a bounded subset D of three dimensional space the conductivity is then

$$\gamma(x) = 1 + (\sigma - 1)\ \chi_D(x).$$

We then take u_0 to be a background field with no object so that $\nabla^2 u_0 = 0$, and consider the perturbed field $u - u_0$ that the fish would measure at its surface:

$$\begin{aligned} \nabla \cdot (\gamma \nabla u) &= 0 \\ u(x) - u_0(x) &= O(1/|x|^{-2}) \text{ as } |x| \to \infty. \end{aligned}$$

Generalized polarization tensor in the scalar case

The full asymptotic expansion of (1) is

$$u(x) - u_0(x) = \sum_{|i|,|j|=1}^{\infty} \frac{(-1)^{|i|}}{i!j!} \partial_x^i \Gamma(x) M_{ij} \partial^j u_0(0)$$

as $|x| \to \infty$. Here i and j are multi indices. The coefficients $M_{i,j}$ of the generalized polarization tensor depend on the shape and conductivity of the target D but not on position. Interestingly the M_{ij} form the coefficients of a symmetric bilinear form on harmonic polynomials [3, Thm 4.10]. Given D and σ one can calculate the M_{ij} by solving integral equations on ∂D and taking moments.

For an isolated target it is relatively easy for a fish (or a mathematician) to locate a point in D. But is it a tasty insect lava or just floating debris? We would like to be able to tell apart different shaped objects. Using a coordinate system where $0 \in D$ we have (from [3, p77])

$$u(x) - u_0(x) = -\nabla\Gamma \cdot M\nabla u(0) + O(1/|x|^2) \qquad (1)$$

where $\Gamma(x) = 1/(4\pi|x|)$ and M is a symmetric matrix that depends on D and σ but *not* on x. It is called a polarization tensor, and for this scalar problem specifically, the Pólya–Szegö tensor. Moreover if D is rotated by an orthogonal matrix R, M becomes $R^T MR$. This means that M can be used to classify the target independently of its position or orientation in space. If a fish could fit a matrix M to its measurements as it swam around to get enough data, it could compare the eigenvalues to those of the M of known prey [5]. The full asymptotic series is given in the box "Generalized polarization tensor in the scalar case", and the complete generalized polarization tensor completely determines D. On the other hand, the lowest term is just a symmetric matrix and one can think of this as the best fitting ellipsoid. If our fish only estimates this, they can determine if the target is long and thin, for example. But this tensor is invariant under reflections so one cannot tell, for example for a conical target, which way the point is facing. A description of what information is contained in the polarization tensors to a certain order is not yet known, but in the two dimensional case there is recent progress using Faber polynomials in [4].

This was a scalar problem and the same results can be applied to the location of objects with a contrast in magnetic permeability (such as ferrous metal) using measurements of static magnetic fields. We now need to extend it to the alternating magnetic fields used in metal detectors, and this is a problem involving vector fields.

The eddy current approximation

The full Maxwell's equations, in the magnetic field H and electric field E are required to describe propagating radio waves such as those used in ground penetrating radar. Metal detectors work at low frequencies at which wavelengths would be kilometres. The eddy current approximation does not allow propagating waves, and in non-conducting areas the magnetic field behaves like a static magnetic field that is complex, while in conductive areas it captures the "eddy currents" induced by the alternating magnetic field and that circulate within the conductor. Let E and H be the (complex) electric and magnetic field, ω the angular frequency, σ the conductivity, μ the permeability and J_0 the current applied (in practice in a coil), then the eddy current approximation is

$$\begin{aligned} \nabla \times E &= \mathrm{i}\omega\mu H \\ \nabla \times H &= \sigma E + J_0 \\ E(x), H(x) &= O(1/|x|), \text{ as } |x| \to \infty. \end{aligned}$$

It is justified in [2] as an asymptotic expansion assuming low frequency and high conductivity.

Before a mathematical theory was derived, it was the opinion of our engineering colleagues that an expression for the magnetic field perturbed by a conductive object would be similar to the scalar magnetostaic case in (1), with a complex matrix multiplying the unperturbed field. Existing theory for the full Maxwell's equations at high frequency predicted that it would be a product with E rather than H. I met with Habib Ammari and Darko Volkov in a cafe in Paris. They are experts on polarization tensors and I explained what was needed for the metal detector problem. The work they produced was surprising. It was a fourth rank tensor contracted with magnetic fields. Perhaps our engineering colleagues were wrong? Paul Ledger and I were later able to show that the rank 4 tensor had symmetries and could be reduced to a symmetric (complex) rank 2 tensor, in a similar way to the Hodge star giving a correspondence between vectors and skew symmetric matrices. Ledger and I have since been able to derive a generalized polarization tensor expansion for the eddy current problem [6].

Next to the mine-field children ride a bike on cleared land

Into practice

In our land mine centre in Manchester, we have a large team working on a prototype mine detector that combines metal detection with ground penetrating radar. Given the importance of clearing land mines, this project is focussed on making a working system as soon as possible and aims at a much higher "technology readiness level" than is normal for academic engineers. The work on the polarization tensor enabled us to design better classification algorithms to distinguish between threat and non-threat objects. We hope in the longer term that the generalized magnetic polarization tensor will provide better characterization, but crucially it exploits the inevitable non-uniformity of the field — using only the second rank tensor we would have preferred fields to be uniform. Ledger and I continue to develop the theory, currently we are looking at the way the tensor components vary with frequency and the commutator of the real and imaginary parts. Our work keep taking us into areas that necessitate learning areas of mathematics new to us.

They asked me if I would like to blow up a mine. They had found a large anti-tank mine and announced there would be an explosion at noon. A charge was set and 200m away I held the detonator. The explosion was bigger than I thought and after a perceptible delay a small fragment of debris on a high trajectory pinged as it made a dent in my camera. At least there was now one fewer mine. As we drove out of the mine clearance project we went through some nearby farmland where two children played on a bike, children who we hoped would inherit safe land to farm.

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Photo courtesy of Sarah Lionheart

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Exact Computations with Approximate Methods

MIGUEL A. MARCO-BUZUNÁRIZ

Computing methods often live in an exact/approximate dichotomy, which can be roughly restated as symbolic/numeric. We show an example of a problem for which both approaches are needed, and give a sketch of how they can be combined. The example consists of the computation of the fundamental group of the complement of algebraic curves.

Introduction

If we were to ask ten random people to give a synonym of the word "mathematics", it is likely that many of them would answer something like "calculations". For a mathematician, that sounds wrong: we know that mathematics is much more than just performing computations. But even at this higher level, we can observe that computing is a fundamental part of mathematical research: some areas, such as numerical analysis or computer algebra, are devoted to developing new computing methods; others, like modelling, make extensive use of computations to solve specific problems; and even in the most abstract areas, like category theory or algebraic topology, the computation of specific examples may be useful to get some insight into the general phenomena that are then formally proved.

There is a big family of computing methods that have been developed over the millenia. Their nature is as varied as the different areas of mathematics that motivated them; but roughly speaking, we can classify them into two big families: the numerical methods, and the symbolic ones. Of course, this classification is an oversimplification: there are cases that could be considered as part of both families, and some probably wouldn't fit in either of them. But the set of cases that fall clearly into one of the two categories is so big, that this oversimplification can be considered accurate in general.

Numerical methods are the ones that deal with numerical approximations of the data (usually by floating point numbers). Hence, they have an intrinsic error associated with them. However, they are in general very fast (at least compared to methods without error). This makes them well suited for problems associated with simulations of physical processes (where even the measurements may have some error, and an approximate solution can be good enough for many purposes). However, they might not be well suited for problems where exactness is a requirement. As good examples of this family, we could choose the methods to find approximate solutions of an equation (such as Newton–Rhapson) or methods to integrate differential equations (such as Runge–Kutta).

On the other hand, symbolic/algebraic methods deal with exact expressions. They are a good fit for problems of an algebraic or geometric nature, where the answers that we look for are often qualitative rather than quantitative (that is, questions like "does this object have this property or not?"), or where approximate solutions don't make sense (consider, for instance, the effect that the determinant of a matrix being exactly zero, or 0.0000001, has on its rank). A non-exhaustive list of well known examples of these methods could be the Cantor–Zasenhaus method to factor polynomials, the LLL method to reduce lattice basis, the Buchberger criterion to compute Gröbner basis, or the Risch algorithm to compute symbolic antiderivatives of functions. But the power of these methods comes at a cost: they often have a high computational complexity. Moreover, as we will illustrate in the following sections, they can be totally hopeless for certain problems.

An algebraic problem that requires numerical methods

In this section we will introduce the problem of computing the fundamental group of the complement of a complex algebraic curve.

Consider a squarefree polynomial $f(x,y) \in \mathbb{C}[x,y]$. The set of points where it evaluates to zero

$$C := \{(x,y) \in \mathbb{C}^2 \mid f(x,y) = 0\}$$

is an affine algebraic curve. If we are interested in the topology of the pair $(\mathbb{C}^2, C)$, then we ought to note the following well known facts:

- The ambient space $\mathbb{C}^2$ is topologically the same as $\mathbb{R}^4$.
- If C contains no critical points of f, its projectivization will be a two-manifold of genus $\frac{(d-1)(d-2)}{2}$ embedded in $\mathbb{CP}^2$, where d is the degree of f.
- If C has singular points, then topologically it will be the result of collapsing some cycles in a smooth deformation of it.

So, to some extent, the topology of both the ambient space and the curve itself are well understood. What can be more complicated is the way that the curve is embedded in the ambient space.

It is a situation analogous to the case of knot theory, where we study how circles can be embedded in $\mathbb{S}^3$. The embedded object and the ambient space are well understood, and the richness of the theory lies in the different ways to embed it.

Also as in the case of knots and links, an object whose topology captures an important part of the complexity of the embedding is the complement $\mathbb{C}^2 \setminus C$. Think of it as "how does the ambient space look after the subspace makes a hole in it". Again, just as in the case of knots, the curve C makes a "hole" of co-dimension 2, so nontrivial phenomena might happen at the level of the fundamental group. That will be our goal in this paper: to compute the fundamental group of the complement

$$\pi_1(\mathbb{C}^2 \setminus C).$$

A general strategy for this computation has been known at least since Zariski, and it was totally formalized by van Kampen. The method works as follows.

Consider the projection to the first coordinate:

$$p: \begin{array}{ccc} \mathbb{C}^2 & \to & \mathbb{C} \\ (x, y) & \mapsto & x \end{array}.$$

For each point $x_0 \in \mathbb{C}$, the preimage $p^{-1}(x_0)$ is a complex vertical line. The intersection of this vertical line with the curve C will be given by the values

$$p^{-1}(x_0) \cap C = \{(x_0, y) \mid f(x_0, y) = 0\},$$

that is, the complex solutions of a polynomial equation in one variable. We know that generically, this set will consist of as many different values as the degree of the polynomial. However, it might happen that some of these values are concentrated in the same point (i.e. the polynomial might have multiple roots). This will happen precisely when the *discriminant* of the polynomial is zero. Since the coefficients of this polynomial are, in turn, polynomials in x_0, the set Δ of values for which the vertical line contains fewer points is also determined by a polynomial in x_0.

So, if we move x_0 in a continuous way, the pre-images $\{y_1, \ldots, y_d\}$ will also move continuously. If x_0 moves to a point in Δ, two or more of the y values will collapse into a single point.

Consider now what happens if x_0 makes a closed loop in $\mathbb{C} \setminus \Delta$. We will have a set of points in the complex plane that move continuously, without ever colliding, and end up in the same position as they started. Note that the fact that the set of final positions coincides with the set of initial positions does not imply that each point y_i describes a closed path: the path might induce a permutation on the set of $y's$.

It can also be seen that, if we deform the loop traced by x_0 in a continuous way (but without ever touching Δ), but keeping the endpoints fixed, the trajectories of the y_s will also be deformed continuously, without ever colliding, and fixing the endpoints. These "multipaths", considered up to continuous deformation that fixes the endpoints, are called *braids*. The concatenation of braids endows this set with a group structure (denoted by B_d); and what we have said in the last two paragraphs can be summarized by saying that we have a group homomorphism

$$BM : \pi_1(\mathbb{C} \setminus \Delta) \to B_d.$$

This morphism is called *braid monodromy* of the curve. It was considered by Zariski for the study of the topology of complex algebraic curves. Van Kampen, who was Zasriski's student at the time, proved that it can be used to compute the fundamental group of the complement of the curve, as follows.

Consider the vertical line $p^{-1}(x_0) \setminus C$. Since it is contained in $\mathbb{C}^2 \setminus C$, we have a morphism of groups

$$\pi_1(p^{-1}(x_0) \setminus C) \to \pi_1(\mathbb{C}^2 \setminus C),$$

so the generators of $\pi_1(p^{-1}(x_0) \setminus C)$ can be seen as elements in $\pi_1(\mathbb{C}^2 \setminus C)$. The braid group of d strands acts on $\pi_1(p^{-1}(x_0) \setminus C)$ by "deforming the punctured plane as it is pushed through" the braids (see figure 1). It can be seen that, given a loop in $\pi_1(p^{-1}(x_0) \setminus C)$, the result of deforming it by a braid in the image of BM must be equivalent to the starting loop when they are both considered as elements of $\pi_1(\mathbb{C}^2 \setminus C)$. What van Kampen proved is that these two ingredients are all we need to determine the fundamental group of the complement of a curve.

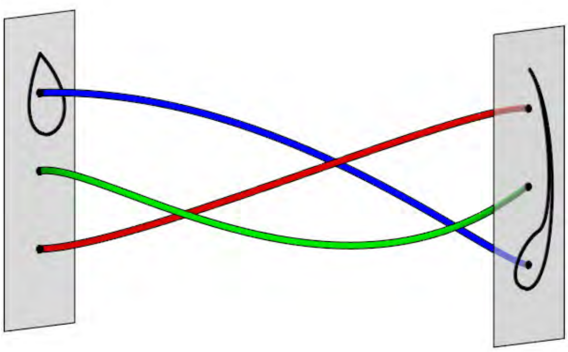

Figure 1. The deformation of a loop by a braid

In particular we have the following:

Theorem 3. *A presentation of the group $\pi_1(\mathbb{C}^2 \setminus C)$ is obtained by the following data:*

- *The generators are given by the generators of $\pi_1(p^{-1}(x_0) \setminus C)$.*
- *The relations are given by $b(x) = x$ for every generator x and every braid b correspoding to a loop in $\pi_1(\mathbb{C} \setminus \Delta)$.*

Summarizing, the braid monodromy is all we need to compute the fundamental group of the complement. Moreover, it is also known that the braid monodromy determines completely the topology of the embedding $C \to \mathbb{C}$. We can encode all the topology we are interested in with just a list of braids. This is clearly a strong motivation to be able to compute these braids.

Arithmetic Zariski pairs

In principle, since the problem we are interested in is given by exact inputs (the polynomial that defines the curve), and requires an exact output (a list of braids, or a presentation of a group, depending on what we are interested in), one would be tempted to look for exact/algebraic methods to compute it. Sadly, as we will see, this is not possible. One reason for that impossibility is the existence of the so-called arithmetic Zariski pairs.

A weak arithmetic Zariski pair is a pair of polynomials (f_1, f_2) whose coefficients are defined over some number field $\mathbb{F}$, such that the following conditions hold:

- There exists a Galois transformation $\mathbb{F} \to \mathbb{F}$ that sends the coefficients of f_1 to the corresponding coefficients of f_2.
- The curves defined by f_1 and f_2 have non-homeomorphic embeddings.

The first condition tells us that, from a purely algebraic point of view, the two polynomials are indistinguishable: every purely algebraic computation that we could perform from f_1 could be exactly replicated, via the Galois transformation, on f_2, and the result would also be related by the same transformation. If the final result is some purely combinatorial data (such is the case for group presentations or lists of braids), where no elements of the field $\mathbb{F}$ appear, it should be exactly the same in both cases.

However, the second condition tells us that the final results could not be the same, at least in the case of the computation of braid monodromies, because equal braid monodromies would imply homeomorphic embeddings. So, should a weak arithmetic Zariski pair exist, it would be impossible to compute its topology with purely algebraic methods. Examples of these pairs have been known for some time (see [1] for a survey of the subject), so we can rule out the use of purely algebraic methods to compute braid monodromies.

Moreover, even arithmetic Zariski pairs, in which the fundamental groups are not isomorphic, are known to exist. Examples of this behaviour were also found in [2]. So even if we could find a method of computing the groups that does not rely on the braid monodromy, we would still need numerical approximations at some point. But, how can we prevent the error that numerical methods introduce from giving us incorrect answers? In the next section we will see a way to do so.

Effective computation of the braid monodromy

If we want to compute the braid monodromy of a curve, the first thing that we need to do is to choose paths in the basis $\mathbb{C} \setminus \Delta$. Since we want to stay away from the points of Δ as much as we can (because that is where the approximation errors are more critical), we can use the segments that form the Voronoi diagram of Δ. Given a finite set of points $\{p_1, \ldots, p_n\}$ in the plane, we can divide the plane into regions, where each region is formed by the points that are closer to p_i than to any other p_j. The separation between adjacent regions are pieces of the perpendicular bisector of the segments joining the corresponding points, so a path that moves along the segments on a Voronoi diagram would maximize the distance to the closest point p_i.

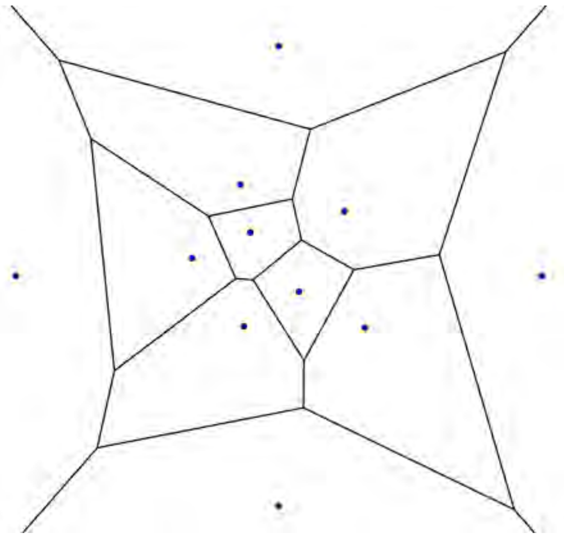

Figure 2. Segments in a Voronoi diagram

In particular, the choice of piecewise linear paths on the basis allows us to reduce the problem of computing the braid monodromy, to computing the braid traced by $p^{-1}(x) \cap C$ when x moves along a segment. By a change of variables, we can assume that the segment joins 0 to 1. The natural way to attack this problem is to take samples $0 = x_0 < x_1 < \ldots < x_l = 1$ and consider in each case the points

$$p^{-1}(x_i) \cap C = \{(x_0, y) \mid f(x_0, y) = 0\}.$$

A linear interpolation of these points should give us an approximation of the path followed by the points, and hence the braid (because we are not interested in the actual path, but just on how the strands cross over each other). However, we need some way to ensure that this piecewise linear approximation we compute is a topologically correct representation of the braid. One way to do so is to ensure that there are mutually disjoint tubular neighbourhoods around our approximations that contain the actual path followed by the roots.

Interval arithmetic and Newton method

The main tool that we will use to ensure the existence of the aforementioned tubular neighbourhoods is the so-called interval Newton method. It relies on interval arithmetic, which is a way to compute with approximate numbers, keeping track of the possible error that computations might introduce. More formally, consider the set of closed intervals

$$\{[a, b] \subseteq \mathbb{R} \mid a \leq b\}$$

and then consider the operations

$$[a, b] + [c, d] = [a + c, b + d]$$

$$[a, b] \cdot [c, d] = \{x \cdot y \mid x \in [a, b] \text{ and } y \in [c, d]\}.$$

Subtraction and division can be defined analogously. The resulting structure is not a ring (possibly lacking even additive inverses), but for many practical purposes we can do computations with these intervals just as we do with real numbers. This same idea can be extended to complex intervals of the form

$$[a, b] + i\,[c, d]$$

although the definition and computation of the operations is a bit more cumbersome. However, it is possible to compute with complex intervals. One way to think of these computations is to imagine that we have some unknown numbers, that are contained in known regions (in this case, rectangles). The computations performed with these rectangles will produce as a result a new rectangle, in which we can be sure that the result of the corresponding computations of the unknown numbers would live.

Given a region R in $\mathbb{C}$, we denote by $[R]$ the smallest complex interval that contains it. The main result that we need is the following:

Theorem 4. *Let Y be a complex interval containing y_0, and let $f : Y \to \mathbb{C}$ be a holomorphic function. Assume that $0 \notin [f'(Y)]$, and set*

$$N(f, y_0, Y) := y_0 - \frac{f(y_0)}{[f'(Y)]} \subseteq Y$$

. Then there exists a unique zero of f in Y. Moreover, this zero lives in $N(f, y_0, Y)$.

This can be seen as an interval version of Newton's method of finding roots, but it has the advantage of giving us a guarantee that a unique solution exists in a certain region.

We will use it in the following way.

(1) Fix $x_0 = 0$, substitute it in f to get f_{x_0}, and find the approximate roots $y_1, \ldots, y_d$.

(2) For each root y_i, estimate an interval Y_0 around it where $\frac{\partial f_{x_0}}{\partial y} \neq 0$.

(3) Apply the interval Newton method to check that there exists a unique root of f_{x_0} in Y. Keep decreasing the size of Y until the test passes.

(4) With a fixed Y, estimate an interval $X = [x_0, x_1]$ where the condition will still hold. Substitute X for x in f to obtain a polynomial f_X with interval coefficients. Apply the interval Newton method to verify that for every $x \in X$, there exists a unique y in Y such $f(x, y) = 0$. If the test fails, keep decreasing X until it passes.

(5) Substitute $x = x_1$ in f, and use the Newton–Raphson method from y_i to obtain an approximation y_i^1 over x_1. Repeat the process from step (2) until we get to $x = 1$.

This method will produce a piecewise linear approximation of the path followed by y given by the points

$$(x_0, y_i), (x_1, y_i^1), \ldots$$

By construction, we know that there is a tubular neighbourhood around this path that contains the actual path followed by the root y. Together with some technical details omitted here, this gives us a guarantee that the piecewise linear paths form a topologically correct model for the actual braid that we want to compute.

This method has been implemented in a library called Sirocco [3], which is included in the computer algebra SageMath [5]. It is able to compute the fundamental group of the complement of a curve in an exact way, even if internally approximate computations are used.

Acknowledgements

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Miguel is a lecturer in mathematics at Universidad de Zaragoza, Spain. His main research interests are in the topology of algebraic varieties, but he also has a keen interest in computer algebra, and is one of the developers of the SageMath computer algebra system.

Reciprocal Societies: Irish Mathematical Society

The Irish Mathematical Society (IMS) is a small, and rather young mathematical society. It was founded in 1976 and has around 350 members. Although many of our members live on the island of Ireland, a good number reside abroad. The IMS aims to foster the development of Mathematics, in particular that with an Irish connection, and it interacts with other mathematical societies through various reciprocity agreements. The Society supports mathematical activities such as conferences and workshops held in Ireland and organised by its members. A list of recent events can be found on the IMS's website. The IMS also plays a role in promoting Mathematics to the general public, emphasising its fundamental importance for modern society.

Traditionally science in Ireland was represented through the national academy, the Royal Irish Academy (RIA), established in 1785 and granted royal charter in 1786, which is based in Dublin. One of the Academy's best-known presidents was Sir William Rowan Hamilton during the period 1837 to 1846. Mathematical activities such as scientific meetings were organised together with the Dublin Institute of Advanced Studies (DIAS), a research institute for Theoretical Physics and Celtic Studies founded by Éamon de Valera in 1940. Upon returning from their PhD studies in the United States, a group of young mathematicians comprising F. Holland, J. Kennedy, D. McAlister and T.T. West started a series of Summer Schools, the first of which took place in July 1969 at Trinity College Dublin, focussing on Group Representations and Quantum Mechanics. At one of the later meetings, a draft constitution of the IMS, based on that of the Edinburgh Mathematical Society, was drawn up by D. McQuillan, J. Lewis and T.T. West. At the DIAS Easter Symposium, this constitution was accepted on 14 April 1976 and thus the Irish Mathematical Society, or by its Irish name, *Cumann Matamaitice na hÉireann* came into being.[1]

The IMS is an all-Ireland professional organisation. One of its main activities is the Annual Meeting (which includes the Society's AGM), usually held in early September or late August. Twice, in 2004 at Queen's University Belfast and in 2009 at the National University of Ireland Galway, it was held jointly with the *British Mathematical Colloquium*. This meeting covers all aspects of Mathematics, pure and applied, as well as Mathematics Education and often attracts well-known speakers from abroad. The *Fergus Gaines Cup* is awarded at this meeting to the best performer in the *Irish Mathematical Olympiad*.

Refrain from 'The Ballad of Rowan Hamilton', from 'Twenty Years of the Hamilton Walk', Ó Cairbre, F., Irish Math. Soc. Bulletin 65 (2010), 33–49

The Society's main publication is the *Bulletin* which appears twice per year. Its aim is to inform Society members, and the mathematical community at large, about the activities of the Society and about items of general mathematical interest. Articles written in an expository style such as informative surveys, biographical and historical articles, short research articles, classroom notes, Irish thesis abstracts, book reviews and letters are included. All areas of mathematics are considered, pure and applied, old and new. The *Bulletin's* first issue appeared in 1978 (then called the *Newsletter*); nowadays all issues are freely available online from the Society's website.

More information about the IMS can be found on its website at maths.tcd.ie/pub/ims/.

Martin Mathieu, MRIA
former President of the IMS

Editor's note: the LMS and the IMS have a reciprocity agreement meaning members of either society may benefit from discounted membership of the other.

[1]In compiling the above historic information I could fortunately draw on an excellent survey by the late T.T. West: *The Origins of the IMS*, Irish Math. Soc. Bull. 51 (2003), 73–75.

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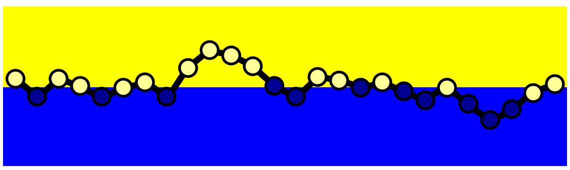
Microthesis: Processes with Constrained Local Time

ADAM BARKER

Studying stochastic processes under certain constraints is important in many applied settings, including polymer physics. Here, we shall restrict the rate at which a process returns to the origin. Then we determine, depending on the strength of the constraint, whether the constrained process is transient or recurrent.

Localised and delocalised polymers

The problem we shall study is related to work in polymer physics. For a polymer made up of two different types of monomer, near a boundary between two fluids, the monomers may be attracted to or repelled by the fluids, and the polymer tries to place as many monomers in their preferred fluid as possible.



Polymer at fluid boundary: blue monomers are attracted to water; yellow monomers are attracted to oil.

Of particular interest is the phase transition between *localisation*, where the polymer remains close to the interface, and *delocalisation*, where it moves away. Typically, the goal is to understand when the transition occurs, as model parameters vary, see [3].

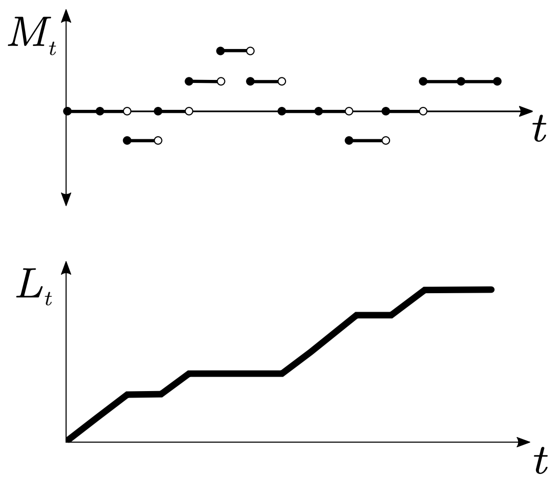
Here, we study random processes constrained to avoid a point which is analogous to the fluid boundary. We will determine exactly how strong a constraint needs to be in order to change the behaviour of a given process from localised to delocalised.

Here, transience and recurrence correspond to delocalisation and localisation, respectively. A Markov process is *transient* if after some finite time, it never returns to 0 (almost surely), and *recurrent* otherwise.

Local time of a Markov process

The process is constrained to avoid a point in a *weak* sense, so the process is still allowed to visit the point, but its rate of return to the point is restricted. This constraint is made rigorous through the notion of *local time*, which we first define in a discrete setting.

The local time process $(L_t)_{t\geq 0}$ of a Markov process $(M_t)_{t\geq 0}$ on a discrete state space records how long is spent at 0 until time t, that is, $L_t := \int_0^t \mathbb{1}_{\{M_s=y\}} ds$.



Local time of a Markov process on a discrete state space

Many Markov processes on a continuous state space, such as Brownian motion, only spend an infinitesimal period of time at any single point. Hence the notion of local time has to be adapted by taking limits:

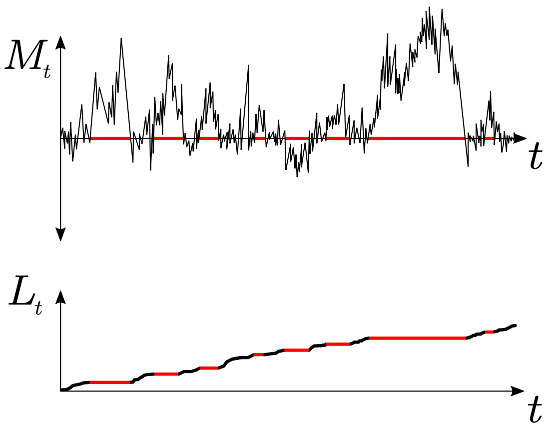
$$L_t := \lim_{\varepsilon\to 0} \frac{1}{2\varepsilon} \int_0^t \mathbb{1}_{\{|M_s|<\varepsilon\}} ds.$$

The local time is a rescaled measure of how long the process spends near 0. We are going to restrict the local time of a recurrent process to grow slower than some function f, in the sense that $L_t \leq f(t)$ for all t.

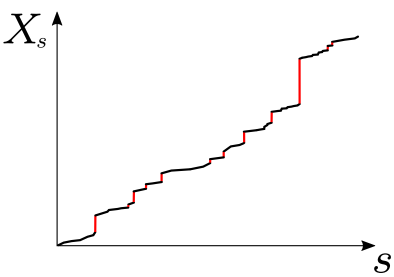
Methodology: Lévy processes

The local time process $(L_t)_{t\geq 0}$ is not usually studied directly. Instead, it is generally best to work with its inverse. The *inverse local time* $(X_s)_{s\geq 0}$ is defined by $X_s := \inf\{u \geq 0 : L_u > s\}$. The inverse local time process has the remarkable property that it always has stationary, independent increments, and is called a *Lévy process* — see [2] for a more detailed account.

The inverse local time process is non-decreasing, and can be thought of as a generalisation of the simple Poisson counting process, whose jumps correspond to *excursions* of $(M_t)_{t\geq 0}$ away from 0.



Local time of continuous Markov process, excursions in red



Inverse of local time, jumps correspond to excursions above

A non-decreasing Lévy process is typically studied using its *Laplace exponent*, ϕ, defined for $t, \lambda \geq 0$ by $\mathbb{E}[e^{-\lambda X_t}] = e^{-t\phi(\lambda)}$. Here ϕ has the general form

$$\phi(\lambda) = \mathrm{d}\lambda + \int_0^\infty (1 - e^{-\lambda x})\Pi(dx),$$

where $\mathrm{d} \geq 0$ is a linear drift, and $\Pi(dx)$ is a measure which determines the size and rate of the random jumps. For our purposes, $\mathrm{d} = 0$, so we aim to formulate our results in terms of the measure $\Pi(dx)$.

Main results: necessary & sufficient conditions

In my work [1], we take a recurrent process, and constrain its local time so that $L_t \leq f(t)$ for all t. We prove that the following necessary and sufficient condition tells us if the constraint is strong enough to change the process to become transient: it is transient (delocalised) if

$$\int_1^\infty f(x)\Pi(dx) < \infty,$$

and the process stays recurrent (localised) otherwise. Somewhat surprisingly, this necessary and sufficient condition can also be understood in terms of the rate of growth of X_s as $s \to \infty$, as it is known that

$$\int_1^\infty f(x)\Pi(dx) < \infty \iff \lim_{s\to\infty} \frac{X_s}{f^{-1}(s)} \overset{a.s.}{=} 0,$$

where the limit holds in an almost sure sense. So the boundary choice of f, at which the process changes from recurrent to transient, coincides with X_s growing to infinity faster/slower than $f^{-1}(s)$.

The constraint $L_t \leq f(t)$ is equivalent to $X_s \geq f^{-1}(s)$, and transience corresponds to $(X_s)_{s\geq 0}$ reaching an infinite value in a finite amount of time. So we have in fact also determined how strong a growth condition can be imposed on the Lévy process $(X_s)_{s\geq 0}$ while still ensuring that it does not explode to infinity. This is important in the general context of developing a deeper understanding of the class of Lévy processes.

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Adam is a PhD student at the University of Reading, supervised by Mladen Savov. Adam's research interests lie in probability theory, with a focus on path properties of Lévy processes and related processes. In his spare time, he enjoys looking after swans and geese.

I is a Strange Loop

Barbican, London, 21 March 2019

Review by Peter Saunders

This year the London Barbican is presenting *Life Rewired*, a season investigating how recent rapid advances in science and technology are affecting our lives. As part of this, the mathematician Marcus du Sautoy curated in March a set of three events with the title *Strange Loops*. (In *Gödel, Escher, Bach*, a review of which can be found in Issue 482 of the *Newsletter*, Douglas Hofstadter defined a strange loop as a cyclic structure that goes through several levels in a hierarchical system in such a way that if we move in one direction through the system we find ourselves back where we started. This is illustrated by Escher's images of two hands drawing each other, or of monks on an ever-ascending staircase.) *I is a strange loop* is a dialogue between two characters, X, a mathematician, and Y, an actor, played in this production by the authors, du Sautoy and Victoria Gould. They deal with a number of issues, but a theme running through the whole play is that while the world of mathematics is related to the real world, sometimes in ways we may not have thought of, in other respects it is fundamentally different.

Thus, for example, X believes he can reach a corner of the cube by taking an infinite number of ever decreasing steps, but when Y challenges him to do this he finds he cannot. He confidently sets out to cut an orange into an infinite number of pieces, but again he fails. He cannot even find infinity in time because nothing, especially a life, lasts forever. But the argument goes in both directions. Y is an actor, and the theatre, like mathematics, has its own rules and conventions. While the audience are meant to imagine there is an infinity of cubes, or at any rate a very large number of them, we know that in reality there is only one. Y claims to go around a finite universe, but we know that all she has really done is exit stage left, nip behind the set to the other side, and enter stage right. Mathematics and the arts have more in common than you may have thought.

Image courtesy of Benjamin Ealovega, Science Museum

If you saw du Sautoy and Gould's earlier play *X&Y*, you will recognise much of it here: two characters called X and Y, a set consisting of a white skeleton cube on a black background, the same sorts of discussions about infinity. But *I is a strange loop* goes well beyond its predecessor, both in terms of the ideas and as a piece of theatre. I was particularly struck by the way it begins with mime, transforms into conventional dialogue, and then ends with mime, similar — but not quite the same — to the way it began. A strange loop, really. I very much enjoyed *I is a strange loop*, and so did the non-mathematician friend who was with me. I'm sure it would appeal to a wide audience, and some of them would even discover there's more to mathematics than they ever imagined.

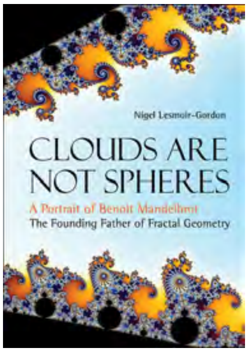
Peter Saunders

Peter is an emeritus professor at King's College, London. His chief research interest is in mathematical biology and he has been active in maths education.

Clouds Are Not Spheres: A Portrait of Benoît Mandelbrot, The Founding Father of Fractal Geometry

by Nigel Lesmoir-Gordon, World Scientific 2018, hardback, pp104. £51, ISBN: 978-1-78634-474-8

Review by Kenneth Falconer



This book celebrates Benoît Mandelbrot's scientific life and the development of fractals with which his name has become synonymous. The opening chapters trace his early years: his birth in Warsaw in 1924, the flight of his family to France with the rise of the Nazis, and the mathematical influence of his uncle, Szolem Mandelbrojt. After the war he enrolled at the École Polytechnique where he studied under Gaston Julia and Paul Lévy. He took a Master's degree in aeronautical engineering at CalTech, before returning to Paris where he completed his PhD in 1952 with a thesis half on Zipf's law and half on thermodynamics. He could have stayed in France with an assured academic career but he was too unconventional for that. After a number of jobs, including a postdoc at Princeton with Von Neumann, he moved to IBM at Yorktown Heights on the outskirts of New York in 1958. IBM suited him, as it gave him the freedom to work largely on what he chose, and there he remained for 35 years.

The book provides an insight into Mandelbrot's conception and development of fractal geometry. At IBM two principal ideas, both of a highly visual nature, were germinating in Mandelbrot's mind which culminated in his seminal work *The Fractal Geometry of Nature* [1] in 1982. The von Koch curve, the Sierpiński triangle and other mathematical constructions were introduced in the late 19th or early 20th century purely to illustrate topological or analytical quirks. Mandelbrot's first insight was that these should not be regarded as isolated pathological examples, but rather as instances of a vast class of objects with common features such as irregularity at all scales, some sort of self-similarity, and a non-integer dimension. Secondly, he suggested that many real phenomena, including clouds (hence the title of the book), topographic surfaces, turbulence, share prices, and even the universe, had an irregularity that shared such features, at least over a range of scales. A new approach was needed to describe, quantify and analyse them. Mandelbrot coined the word 'fractal' for such objects, both theoretical and real, from the Latin 'fractus' meaning 'broken'. He freely admitted that choosing this succinct yet suggestive name played a major role in establishing fractals as a new area of study that was both unified and diverse.

Of course, there is a chapter on the Mandelbrot set, including an account of its 'discovery'. The book portrays the excitement of Mandelbrot and his colleagues as they saw increasingly detailed images of the set and as its intricacy and significance became apparent. The set attracted enormous interest not only from mathematicians and scientists, but also from the public who could appreciate artistically the almost fantastical pictures, often coloured according to escape time. The pictures in the book will impress, though readers not already familiar with the Mandelbrot set and the associated Julia sets may find their construction hard to fathom from the text (not helped by typographical inconsistencies in the few equations that there are).

The book includes quotes from many who, in the 1980s, enthusiastically embraced the emergence of fractals, though there were skeptics. I first met Mandelbrot in 1984 at a workshop at Les Houches in the

French Alps. He was charming, and fortunately liked my talk on the 'digital sundial theorem' — although essentially about topology and measure, he appreciated the visual interpretation. He was charismatic, but also very possessive of his ideas and work. Sometimes young researchers were reduced almost to tears for daring to challenge him. But once his ideas were broadly accepted he mellowed somewhat — in 1999 he played a key role in the Isaac Newton Institute programme on 'Fractals and Applications' where he interacted positively with all the other participants.

Benoît Mandelbrot at the Newton Institute in 1999. (Image courtesy of the Isaac Newton Institute.)

Mandelbrot was a polymath, at different times working on finance, data transmission, turbulence, computer graphics, meteorology and many other areas, and the common roughness he observed in such a diversity of settings was important in developing the fractal concept. Despite his original training he did not regard himself as a mathematician, indeed he considered much of pure mathematics, including the formal mathematics of geometric measure theory often used in studying fractals, as 'dry'. Nevertheless his deep insights have led to dramatic progress in mathematics. After a numerical study he conjectured that the exterior boundary of a planar Brownian trail had Hausdorff dimension exactly $\frac{4}{3}$, a conjecture eventually proved in 2000 by Greg Lawler, Oded Schramm and Wendelin Werner, who in the process pioneered the theory of conformal invariance of random curves and Schramm–Loener evolution. Again, the Mandelbrot set led to a resurgence of interest in complex dynamics and stunning progress by leading mathematicians. There remain many unsolved problems on the geometry of fractals that occupy the attention of top researchers today.

There is more public awareness of fractals and the Mandelbrot set than of many modern scientific concepts and this story of the life and work of the 'Father of Fractals' will interest a lay as well as a professional audience. The book is well-illustrated, earlier chapters with monochrome portraits of Mandelbrot, his family and those who influenced him, and later ones with striking colour pictures not only of the Mandelbrot set and other computer generated fractals, but also of 'real' fractals including cloud formations and rural and mountain scenes; sadly some of the plates lack captions (not all readers will recognise the two photographs of Buachaille Etive Mor in Glencoe!).

This celebration Mandelbrot's scientific life is largely based on interviews that the author had with him when making films on his work. Shortly after his death in 2010, Mandelbrot's autobiography *The Fractalist: Memoir of a Scientific Maverick* [2] was published. Both of these books derive from Mandelbrot's accounts and have a similar viewpoint. There was controversy in Mandelbrot's work not mentioned in either, for example, Brooks and Matelski's coarse but recognisable picture of the Mandelbrot set in 1978, two years before Mandelbrot's images. A challenge for historians of mathematics and science in coming years will be to produce a more broadly contextual and rounded account of the advent of fractals.

FURTHER READING

[1] Benoit B. Mandelbrot, The Fractal Geometry of Nature, W.H. Freeman, 1982.
[2] Benoit B. Mandelbrot, The Fractalist: Memoir of a Scientific Maverick, Panthenon Books, 2012.

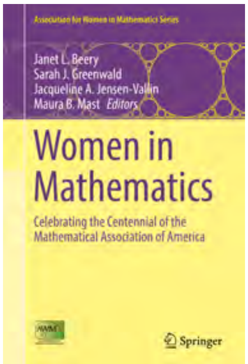
Kenneth Falconer

Kenneth has been professor at the University of St Andrews since 1993 where he was recently appointed Regius Professor of Mathematics. He works mainly on topics that are somehow related to fractals, about which he has written four books. He enjoys hillwalking, has twice climbed all the Munros, and goes on continuous 100 mile cross-country walks.

Women in Mathematics: Celebrating the Centennial of the Mathematical Association of America

ed. by J.L. Beery, S.J. Greenwald, J.A. Jensen-Valin and M.B. Mast, Springer, 2017, hardback, pp405. £88, ISBN: 978-3-31966-6938

Review by Sarah Hart



As part of the 100th anniversary celebrations for the Mathematical Association of America, the Association for Women in Mathematics sponsored a themed paper session at MathFest 2015 entitled *The Contributions of Women to Mathematics: 100 Years and Counting*. This volume, consisting of 21 papers by 26 authors, was one outcome from that session.

Given the origin and title of the book, it is no surprise that a large proportion of the content focuses on women mathematicians working in the United States. Indeed, all but two of the 26 authors are based in America. However, there is still a great deal which will be of broader interest. The editors have done a good job in selecting and grouping the papers into three strands — there are biographical papers, papers on mathematics education, and beginning the book, with the largest section, thematic papers, on women mathematicians in particular times or places. Thus, the first part of the book contains chapters on Girton women from 1870 to 1940, women gaining PhDs in America from the 1880s to the 1950s, Polish women mathematicians, Canadian women mathematicians and so on. In every chapter, around the plain historical facts there are fascinating nuggets to be found, many inspiring stories and not a few reminders of how far we have come in the last century, in terms of attitudes to women in mathematics (not far enough, of course).

The chapters on women PhDs in the US and on Black American women mathematicians, summarise books and websites created by Judy Green, Jeanne LaDuke, Erica Walker and Margaret Murray in which biographical information has been collected on all 419 women who gained PhDs in mathematics in the United States from the first (Winifred Edgerton Merrill) in 1886, up until 1959. Interestingly, it turns out that in the first 100 years of women earning mathematics PhDs in America, women earned over 3000 PhDs, which was over 10 % of the total.


Grace Chisholm Young

This is higher than many of us would have guessed. In fact, by the 1930s the annual proportion was nearing 15%, but there appears to have been a kind of post-war backlash where women were expected to get back to the kitchen and let men have the "real" work again. There are anecdotes to illustrate this: one woman tells of how she was finally hired in an academic post, only to be telephoned two days later by the Head of Department to say that he had heard of a young man who really needed a job and so he had given her job to him — "I'm sure you understand", he said. The chapter on Black American women mathematicians was an excellent exploration of the history of Black women's contribution to mathematics in the United States.

For comparison, less seems to have been explicitly written about "firsts" in women's mathematics PhDs in Britain, though we do have the extensive database created by A.E.L. Davis which catalogues all the 2500 women who gained a first degree in mathematics between 1878 (when women were first admitted) and 1940. It took longer for PhDs to start being awarded in the UK than in continental Europe and elsewhere.


Dorothy Wrinch

Grace Chisolm Young gained a Ph.D. in 1896 from Gottingen — was she the first British woman to gain a mathematics PhD? This must be known, but it's certainly not as easy to find the information online as it is to find information about American women PhDs. Similarly, who was the first woman awarded a mathematics PhD (or similar) from a British University? Was it Dorothy Wrinch, who was awarded a D.Sc. in 1921 from the University of London? Who was the first Black or minority ethnic British woman to gain a mathematics PhD? Answers to the corresponding questions for American mathematicians are readily available online.

Part II of the book deals with biographies of individual women, opening with Noel-Ann Bradshaw's chapter on Florence Nightingale, who fortunately disregarded the advice of prominent statistician William Farr that statistics ought to be "the dryest [sic] of all reading". It is good to be reminded again of the power and clarity of excellent diagrams, and the huge impact of Nightingale's work on reducing mortality rates. Other chapters in this part of the book look at the lives and careers of Emmy Noether, Constance Marks, Mina Rees, Gertrude Cox and Norma Hernandez.

The third and final part of the book consists of papers relating to teaching, education and outreach. Among other things, there are chapters on excellent programmes such as EDGE (Enhancing Diversity in Graduate Education), and interviews with two women (Jackie Dewar and Sarah Greenwald) who have successfully run university lecture courses on women and mathematics. Both women have made many resources freely available online, and members may be interested to visit their websites: tinyurl.com/jdewar; and tinyurl.com/y6kbvmhu.

For me, a highlight of this final section of the book was the chapter on using humour to combat inequities, which described a series of skits highlighting common "micro-inequities" experienced by women in mathematics, in order to raise awareness.


Winifred Edgerton Merrill

The term micro-inequity was used by the Mathematical Association of America's Committee on the Participation of Women, to describe those unfair and/or offensive behaviours that individually may seem too small to be damaging, but when they are repeatedly experienced can contribute to making an environment hostile. Examples from my own experience (that I'm sure have happened to nearly all women in mathematics) include people assuming, at a conference social event, that you are there because you are someone's wife rather than because you are a mathematician (even if that assumption happened to be correct, it's still better to open with "are you a mathematician?" than "who are you here with?"); people asking if you are working towards a PhD (it's embarrassing all round when you have to say "well not really, I'm a professor") and so on. Overt sexism is thankfully now very rare, but statements arising from tacit assumptions based on gender can still send the "you don't fit" message. Tackling this kind of thing with humour, as the skits presented in the chapter do, is a great way to raise awareness in an inclusive, non-aggressive way.

In summary, I found this book to be inspirational and informative. I think it has a place in every university library.

Sarah Hart

Sarah is Professor of Mathematics at Birkbeck College, University of London, and Head of the Department of Economics, Mathematics and Statistics. Her favourite hobby is embarrassing her children in public by saying things like "look girls, a hyperbolic paraboloid", and "ooh, let's get bus 257, that's one of my favourite Fermat primes".

Obituaries of Members

Edward Fraenkel: 1927–2019

Edward Fraenkel, FRS who was elected a member of the London Mathematical Society on 17 June 1971, died on 27 April 2019, aged 91.

Edward was born in Germany, the youngest child of the eminent classicist Eduard Fraenkel. When Eduard was dismissed from academic posts because of antisemitic laws the family fled Nazi Germany, and settled in Oxford when he became Corpus Professor of Latin in 1935. (In later life Edward quoted[1] his mother Ruth, whom he adored, as saying "Thanks to Hitler we fell upstairs"). Young Edward attended the Dragon School where he was a contemporary of Peter Swinnerton-Dyer who was two months younger. While the rest of his family stayed in the UK during World War II, Edward was evacuated to Canada where he entered the University of Toronto to study aeronautical engineering at the age of 16. Four years later he had completed a postgraduate masters degree with a thesis on the design of nozzles for supersonic wind tunnels.

After the war he returned to the UK and spent four years at the Royal Aircraft Establishment as a Scientific Officer followed by a year in Glasgow before moving to Aeronautical Engineering at Imperial College. Increasingly drawn to mathematics by a desire to put theoretical aeronautics on a rigorous footing, and after an enjoyable stay at Caltech during 1957-8, he moved to the Mathematics Department at Imperial in 1961. By this time his reputation was growing and, after considering several attractive offers, in 1964 he accepted an invitation from George Batchelor to join the Department of Applied Mathematics and Theoretical Physics in Cambridge. From there he moved to Sussex University, which was then emerging as the major centre for theoretical partial differential equations research in the UK[2], first as a research professor for three years in 1975 and then as a member of staff in 1978. Papers combining mathematical rigour with physical relevance remained his forte, but now he was supported and appreciated by colleagues who shared his interests in modern analysis, and he thrived. After semi-retirement he moved in 1987 to the University of Bath where he continued to do important new work and wrote his book *An Introduction to Maximum Principles and Symmetry in Elliptic Problems* (CUP, 2000) on the maximum principle in the theory of elliptic equations. He was awarded the London Mathematical Society Senior Whitehead Prize in 1989 and elected FRS in 1993.

Edward was quite a character, offering cash prizes for solutions to questions he raised during seminars (bigger prizes for students than for professors). He said that to be a good teacher you had to be a bit stupid, for otherwise you couldn't understand the difficulties students were facing. In his celebrated paper[3] on rooms and passages he thanked Louis Nirenberg "for being kind enough to play with good grace the role of Wedding-Guest to my Ancient Mariner on the subject of irregular boundaries". He was an entertaining raconteur and everyone who knew him will remember his posh voice either offering penetrating criticism or self-deprecation, but always with grace and charm.

All his life he kept very fit, with long cycle rides up and down steep hills (but not without serious accidents) into his 70s and skiing enthusiastically into his 80s. He was very interested in the encouragement of young people, through the UK Mathematics Trust, the mathematical olympiad movement and the Scottish Mathematical Council, and made significant personal donations to support individual students.

He is survived by his wife Beryl, his daughters, and grand and great-grand children.

Simon Phillips Norton: 1952–2019

Rob Curtis writes: Simon Norton was quite simply a mathematical phenomenon. Much has been written since his sad premature death in February about his prodigious talent: he represented the UK in the Mathematical Olympiad on three separate occasions, the first when he was just 15 years old, obtaining gold medals on each occasion with higher marks than the strongest Chinese and Russian contenders; he

[1] The Pelican Record, Corpus Christi College, Vol. L, Dec. 2014, p.18.
[2] PJ. Bushell and D.E. Edmunds, The evolution of PDEs in the UK, LMS Newsletter, 475 (2018) 36–38.
[3] L. E. Fraenkel, On Regularity of the Boundary in the Theory of Sobolev Spaces, Proc. Lond. Math. Soc., 39 (1979) 385–427.

obtained a first class honours degree in Mathematics from London University while still a schoolboy at Eton College; and at the age of 17 he entered directly into the third year of the Mathematics Tripos at Cambridge, again obtaining a first class degree.

Following Part III of the Tripos, Simon embarked in 1973 on a PhD under the supervision of John H. Conway. Initially the subject of his thesis was to have been the surreal numbers which Conway had invented in 1969 and still considers to be his most important discovery. Indeed, both Conway's *On Numbers* and *Games and Winning Ways* make frequent reference to Simon's contributions, and the game SNORT was of course invented by S. Norton. However, in 1973 Conway and I had started work on the *Atlas of Finite Groups* and Simon gravitated to my office, later christened *Atlantis*, where the two of us were busy studying various groups. At first it is fair to say that we were somewhat irritated by the omnipresence of this uninvited guest, but within a week or two we realised just how invaluable his contribution was becoming and we asked him to become a co-author, later to be joined by Richard Parker and Robert Wilson.

At around this time Bernd Fischer had predicted the existence of what was to become known as the Monster group M, and for convenience the centralizer of a class of elements of order five was known as F, whilst the centralizer of a class of elements of order three was known as T. The conjectured group F was studied extensively by Koichiro Harada, but a group is not known to exist until it has been constructed, either as permutations or matrices. For the large sporadic groups this was invariably done by computer but Simon, in a brilliant piece of work which became his PhD thesis, constructed F by hand as the group of automorphisms of a certain graph whose vertices and joins he defined in terms of permutations of 12 letters. These days F is known as the Harada–Norton group HN; somewhat fortuitously T is the Thompson group.

In 1978 John McKay noticed that 196883, the degree of the smallest irreducible complex representation of M, is one less than the coefficient of the linear term in the Fourier expansion of the so-called j-function in modular function theory, and it was soon observed that subsequent coefficients were also simple integral combinations of the irreducible degrees. Conway and Norton called this unexpected and remarkable connection between sporadic groups and number theory Monstrous Moonshine and they made a profound conjecture which was finally proved by Richard Borcherds in 1998. It is probably true that Simon knew this extraordinary group in greater detail than anyone else.

In the early days Simon's lectures at conferences were somewhat opaque as he tended to home in on the most subtle and intractable parts of his work without explaining the context. Later he developed a more relaxed and authoritative style, and his talks became informative and enlightening.

Outside mathematics, Simon's passion was public transport. His knowledge of Britain's network of buses was staggering: at one time he could give you the number of the bus route between any two towns in the country. He was a tireless campaigner, establishing and providing financial support for the Foundation for Integrated Transport, which will be a lasting legacy. And he made full use of the transport system himself, meticulously planning lengthy treks around the country which often included buses which only ran once a week to and from the local market.

Unsurprisingly Simon excelled at both bridge and chess, with games he had played featuring in columns of the national press. More unexpectedly we learnt at his funeral that as a child Simon was not only a mathematical prodigy but he also possessed a phenomenal musical gift. He was indeed multi-talented.

Simon Norton was a brilliant, unworldly, unassuming and modest man who inspired affection in those who knew him well. He will be sadly missed.

Walter Craig: 1954–2019

Walter Craig, who was elected a member of the London Mathematical Society on 17 May 1991, died on 18 January 2019, aged 65.

Catherine Sulem and Gene Wayne write: Born on 28 November 1953 in State College Pennsylvania, where his father, the logician William Craig, was a professor in the mathematics department at Penn State, Walter Craig was one of the leading analysts and applied mathematicians of his generation. He received an A.B. and M.S. in Mathematics from UC Berkeley, and a PhD under the supervision of Louis Nirenberg from the Courant Institute (NYU) in 1981. After positions at CalTech and Stanford, Walter moved to Brown

University in 1988. In 2000, Walter, his wife Deirdre Haskell and their daughter Zoe moved to Canada. Walter was offered a Tier 1 Canada Research Chair in Mathematical Analysis and its Applications, the very first appointment through the CRC program in the country.

Walter was renowned for his work on nonlinear partial differential equations, Hamiltonian dynamical systems and their applications, in particular to fluid dynamics and mathematical physics. He had broad mathematical and scientific interests and was skillful at communicating with people from different scientific backgrounds. His work was recognized by numerous prestigious awards including an Alfred P. Sloan Fellowship, an NSF Presidential Young Investigator Award and a Killam Research Fellowship. He was named a Fellow of the Royal Society of Canada in 2007, Fellow of the AAAS in 2008 and was among the inaugural class of Fellows of the AMS in 2013. In 2013 he was appointed Director of the Fields Institute. His contributions at McMaster University were recognized with conferral of the title Distinguished University Professor in 2016.

In addition to being a brilliant researcher, Walter was also an exceptional teacher and mentor who educated and supervised many undergraduate and graduate students, postdoctoral fellows and young mathematicians. Many of them have gone on to become accomplished scholars in various countries around the world.

Walter was a constant source of inspiration, whose enthusiasm and friendship have never waned. He will be greatly missed by all who had the privilege of knowing him as a mathematician, colleague and dear friend.

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Algebraic Spline Geometry Meeting

Location: Swansea University
Date: 21–23 August 2019
Website: tinyurl.com/y4oc5cnp

The aim is to bring together researchers with interests falling broadly under applied algebraic geometry, commutative algebra and multivariate spline approximation, with an outlook towards practical problems arising in areas of application. Supported by the LMS and the Computational Foundry project at Swansea University.

Applications of Mathematics in Ecology

Location: University of Kent
Date: 2 September 2019
Website: tinyurl.com/yy8ay9gs

Mathematical ecology is an ever evolving field requiring collaboration between mathematicians, statisticians, and ecologists. This meeting, supported by a Scheme 9 LMS grant, will discuss new research directions led by contributions from D. Bearup (UoK), N. Petrovskaya (UoB) and J. Potts (UoS). Further details and booking via the website.

Recent Trends in Stochastic Analysis and Partial Differential Equations

Location: University of Chester
Date: 5–6 September 2019
Website: tinyurl.com/y6otert8

This workshop aims to bring together researchers from stochastics and applied mathematics to discuss novel mathematical methods on phase separation. Supported by an LMS Conferences Grant. For further information contact the organisers Dimitra Antonopoulou (d.antonopoulou@chester.ac.uk) and Georgia Karali (gkarali@uoc.gr).

British Logic Colloquium 2019

Location: University of Oxford
Date: 5–7 September 2019
Website: tinyurl.com/y3oyq559

This is the annual meeting of the British Logic Colloquium covering all areas of mathematical logic including philosophy of mathematics and applications to computer science. Invited speakers: Michael Benedikt, Paola Bruscoli, Ehud Hrushovski, Franziska Jahnke, Johannes Stern and Philip Welch. Supported by an LMS Conference grant.

One Day Function Theory Meeting

Location: De Morgan House, London
Date: 2 September 2019
Website: tinyurl.com/y5cxgs93

This annual event brings together established researchers and younger mathematicians, from across the UK, working in the areas of Complex Analysis and Function Theory. It covers a broad range of topics including potential theory, holomorphic dynamics and complex differential equations. Supported by an LMS Conference grant.

Inverse Problems from Theory to Application

Location: University College London
Date: 4–6 September 2019
Website: tinyurl.com/y68jk8qq

This conference will bring together mathematicians and statisticians working on theoretical and numerical aspects of inverse problems, and engineers, physicists and other scientists working on challenging inverse problem applications. Industrial representatives and academics at all levels are welcome.

Water Waves — Mathematical Theory & Applications

Location: University of Plymouth
Date: 5–6 September 2019
Website: tinyurl.com/y59mm2lo

A two-day workshop on theoretical and applied aspects of water waves, touching on a variety of topics from wave kinetic equations to wave-structure interaction and experiments. Slots for contributed talks, including short talks by graduate students, are available; email raphael.stuhlmeier@plymouth.ac.uk for details. Supported by an LMS Scheme 9 grant.

Proof Society Summer School 2019

Location: Swansea University
Date: 8–11 September 2019
Website: tinyurl.com/proofsocSchool19

This event is collocated with the Proof Society Workshop. Topics are universal proof theory, ordinal analysis, structural proof theory, proof mining, functional interpretation, reverse mathematics, Martin-Löf type theory, and bounded arithmetic. Limited grants for UK PhD and Research Masters Students are available. Deadline: 8 July 2019.

LMS Meeting

LMS Prospects in Mathematics Meeting

Lancaster University, 6–7 September 2019

Website: tinyurl.com/yysds39f

All final-year maths undergraduates and masters students who are considering applying for a maths PhD in 2020 are invited to attend the 2019 LMS Prospects in Mathematics Meeting. There are 50 places are available. See website for details on overnight accommodation and funding for travel costs. To apply, email Dr Nadia Mazza (n.mazza@lancaster.ac.uk) with the subject heading 'Prospects 2019 Application' and the statement: "I am on track academically to begin Ph.D. studies in 2020", with evidence of your predicted degree classification. Application deadline: 19 July 2019.

Curve Counting Theories and Related Algebraic Structures

Location: University of Leeds
Date: 9–11 September 2019
Website: tinyurl.com/y5k79u5t

This meeting is devoted to various algebraic structures developed to describe the topological invariants of the moduli spaces of stable curves and their generalizations. There will be up to 17 Invited Speakers. The deadline for registration is 9 August 2019. Supported by an LMS Conference grant.

The Geometry of Derived Categories

Location: University of Liverpool
Date: 9–13 September 2019
Website: tinyurl.com/y4mg7rv6

The aim of this workshop is to exchange and explore developments and ideas on derived categories, their fundamental structure, and their applications to algebraic geometry. Registration deadline is 15 July 2019. The workshop is supported by an LMS Conference grant.

LMS Meeting

LMS Midlands Regional Meeting & Workshop

University of Nottingham, 11 September 2019

Website: tinyurl.com/y4vkvhld

The speakers at this meeting will be Nina Snaith (Bristol University, *Random matrix theory, the Riemann zeta function and elliptic curves*), John Coates (University of Cambridge, *Non-Vanishing of L-values and the exact Birch-Swinnerton-Dyer formula*), and Mark Pollicott (Warwick University, *Dynamical zeta functions and their applications*). The lectures are aimed at a general mathematical audience and all are welcome to attend, including LMS non-members. The meeting forms part of the Midlands Regional Workshop on *Zeta Functions in Number Theory and Mathematical Physics* (11–13 September 2019).

Limited travel support is available: email Chris Wuthrich (christian.wuthrich@gmail.com) for details. Visit the website for further details about the meeting and to register for a place. A Society Dinner will be held after the meeting at the Victoria in Beeston, at a cost of £25.00 including drinks. To reserve a place at the dinner, email Chris Wuthrich.

Proof Society Workshop 2019

Location: Swansea University
Date: 11–13 September 2019
Website: tinyurl.com/proofsocWS19

This 2nd Workshop of the Proof Society, is collocated with the Proof Society Summer School. The goal is to support the notion of proof in its broadest sense, reaching out to all scientific areas which consider proof as an object in their studies, including proof theory. On-campus accommodation is available on a first-come-first-serve basis. The workshop is supported by the LMS. Deadline: 15 July 2019.

Flood Risk Conference

Location: Swansea University Bay Campus, Wales
Date: 12–13 September 2019
Website: tinyurl.com/yyu9gxlk

This 4th IMA conference on Flood Risk will be of interest to flood defence practitioners and managers, statisticians, mathematicians, and civil engineers. Invited Speakers are Professor Michael Bruen (University College Dublin, Ireland) and Professor Guoqing Wang (Nanjing Hydraulic Research Institute, China).

Random Matrices and Applications

Location: University of Sussex
Date: 16 September 2019
Website: tinyurl.com/y6qt3z7g

This one day meeting will discuss recent developments in random matrix theory and its applications in number theory, statistical mechanics and integrable systems. The meeting is part of a broader three day conference celebrating the new appointments of mathematicians at the University of Sussex. Supported by an LMS Scheme 9 grant.

Branching Processes and Their Applications

Location: University of Sussex
Date: 17 September 2019
Website: tinyurl.com/y6qt3z7g

This one-day meeting will comprise four talks, given by researchers with background in branching processes discussing their recent works on random trees, fragmentation processes and random walks. The meeting is part of a three-day event celebrating the new appointments of mathematicians at the University of Sussex. The conference is supported by an LMS Scheme 9 grant.

Groups and Representation Theory Conference in Memory of Kay Magaard

Location: University of Warwick
Date: 11-13 September 2019
Website: tinyurl.com/yy9okkfg

This conference is to celebrate the memory of our colleague Kay Magaard. The conference will cover a number of areas on Group Theory and applications in which Kay was active. These include subgroup structure and modular representation theory of Finite Chevalley groups and the revision of the Classification of Finite Simple Groups.

Functor Categories for Groups

Location: University of Lincoln
Date: 16 September 2019
Website: tinyurl.com/y7wcpmn6

This is the ninth meeting of the Research Group Functor Categories for Groups (FCG). The event starts at 1.15 pm and ends at 4.50 pm. The speakers are Dan Segal, Marialaura Noce and Evgeny Khukhro. The FCG Research Group is supported by an LMS Joint Research Groups in the UK Scheme 3 grant.

British Topology Meeting

Location: University of Warwick
Date: 16–18 September
Website: tinyurl.com/y5b5ums4

The annual British Topology Meeting is returning for its 34th iteration. The aim is to bring together the cohort of topologists in the UK, established and earlier career alike, to discuss and present new and interesting results. Visit the website to register and for more details. We will accept a small amount of contributed talks.

Randomness, Symmetry and Free Probability

Location: University of Sussex
Date: 18 September 2019
Website: tinyurl.com/y6qt3z7g

This is part of a three-day meeting in probability and mathematical physics, supported by an LMS Scheme 9 grant. Speakers on this day are: Natasha Blitvic (Lancaster), Antoine Dahlqvist (Sussex), Mylène Maïda (Lille) and Jon Warren (Warwick). Visit the website for further details.

Philip Maini's 60th Birthday Workshop

Location: Mathematical Institute, Oxford
Date: 18–19 September 2019
Website: tinyurl.com/yyeqzlaf

This meeting celebrates the 60th birthday of Philip Maini, an internationally leading researcher in mathematical biology for decades. The workshop *On Growth and Pattern Formation* will include talks from epidemiology to brain modelling to the mathematics of football. Registration deadline is 5 July; some travel support will be available.

Lattice Polytopes, With a View Towards Geometry and Applications

Location: ICMS, University of Edinburgh
Date: 18–21 September 2019
Website: tinyurl.com/polytopes19

This workshop will focus on the combinatorics of lattice polytopes, with a view towards connection to algebraic geometry via toric geometry as well towards applications. Apply by 2 August for limited graduate student and postdocs support.

Dependence Modeling and its Applications in Insurance and Finance

Location: Heriot-Watt University
Date: 10 October 2019
Website: tinyurl.com/y3y7dfxz

This half-day workshop shares some recent research results on dependence modeling and its applications in insurance and finance. For further details email j.yao@hw.ac.uk.

Southampton–Bielefeld Geometric Group Theory Meeting

Location: University of Southampton
Date: 18–20 September 2019
Website: tinyurl.com/BiSh2019

The general theme of the second joint meeting is Geometric Group Theory, more specifically infinite groups in the presence of non positive curvature. It is open to all who are interested. Supported by an LMS Conference grant. Visit the website for more details and to register.

LMS Popular Lectures

Location: University of Birmingham
Date: 19 September 2019
Website: lms.ac.uk/events/popular-lectures

The LMS Popular Lectures are free annual events, open to all, which present exciting topics in mathematics and its applications to a wide audience. The speakers for the 2019 Popular Lectures are Professor Peter Higgins (University of Essex) and Professor Alain Goriely (University of Oxford).

Complex Analysis in Mathematical Physics and Applications

Location: INI, Cambridge
Date: 28 October – 1 November 2019
Website: tinyurl.com/yyluydkr

This workshop will bring together varied contributors in complex analysis and algebraic geometry. Deadline for applications: 28 July 2019.

LMS Meeting

Annual General Meeting of the LMS

Goodenough College, Mecklenburgh Square, Holborn, London WC1N 2AB, 29 November 2019; 3.00 – 6.00 pm

Website: lms.ac.uk/events/meeting/agm

The speaker will be Marc Lackenby (Oxford). The LMS President, Caroline Series (Warwick) will give the presidential address. The meeting will include the presentation of certificates to all 2019 LMS prize winners and the announcement of the annual LMS election results.

The meeting will be followed by a reception, which will be held at Goodenough College, and the Society's annual dinner at 7.30 pm. The cost of the dinner will be £58.00, including drinks. To reserve a place at the dinner, please email AnnualDinner_RSVP@lms.ac.uk.

For further details about the AGM, please contact Elizabeth Fisher (lmsmeetings@lms.ac.uk).

William Benter Prize in Applied Mathematics 2020

Call for NOMINATIONS

The Liu Bie Ju Centre for Mathematical Sciences of City University of Hong Kong is inviting nominations of candidates for the William Benter Prize in Applied Mathematics, an international award.

The Prize

The Prize recognizes outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, financial, and engineering applications.

It will be awarded to a single person for a single contribution or for a body of related contributions of his/her research or for his/her lifetime achievement.

The Prize is presented every two years and the amount of the award is US$100,000.

Nominations

Nomination is open to everyone. Nominations should not be disclosed to the nominees and self-nominations will not be accepted.

A nomination should include a covering letter with justifications, the CV of the nominee, and two supporting letters. Nominations should be submitted to:

Selection Committee
c/o Liu Bie Ju Centre for Mathematical Sciences
City University of Hong Kong
Tat Chee Avenue, Kowloon, Hong Kong

Or **by email to: lbj@cityu.edu.hk**

Deadline for nominations: 30 September 2019

Presentation of Prize

The recipient of the Prize will be announced at the **International Conference on Applied Mathematics 2020** to be held in summer 2020. The Prize Laureate is expected to attend the award ceremony and to present a lecture at the conference.

The Prize was set up in 2008 in honor of Mr William Benter for his dedication and generous support to the enhancement of the University's strength in mathematics. The inaugural winner in 2010 was George C Papanicolaou (Robert Grimmett Professor of Mathematics at Stanford University), and the 2012 Prize went to James D Murray (Senior Scholar, Princeton University; Professor Emeritus of Mathematical Biology, University of Oxford; and Professor Emeritus of Applied Mathematics, University of Washington), the winner in 2014 was Vladimir Rokhlin (Professor of Mathematics and Arthur K. Watson Professor of Computer Science at Yale University). The winner in 2016 was Stanley Osher, Professor of Mathematics, Computer Science, Electrical Engineering, Chemical and Biomolecular Engineering at University of California (Los Angeles), and the 2018 Prize went to Ingrid Daubechies (James B. Duke Professor of Mathematics and Electrical and Computer Engineering, Professor of Mathematics and Electrical and Computer Engineering at Duke University).

The Liu Bie Ju Centre for Mathematical Sciences was established in 1995 with the aim of supporting world-class research in applied mathematics and in computational mathematics. As a leading research centre in the Asia-Pacific region, its basic objective is to strive for excellence in applied mathematical sciences. For more information about the Prize and the Centre, please visit ***https://www.cityu.edu.hk/lbj/***

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LIU BIE JU CENTRE FOR MATHEMATICAL SCIENCES

Society Meetings and Events

July

1–5 LMS Research School: *Random Structures: from the Discrete to the Continuous,* Bath (482)

8–12 LMS Research School, *Mathematics of Climate,* Reading

September

6–7 Prospects in Mathematics Meeting, Lancaster (483)

11 Midlands Regional Meeting, Nottingham (483)

19 Popular Lectures, University of Birmingham (483)

November

21 Joint Meeting with the Institute of Mathematics and its Applications, Reading

29 Graduate Student Meeting, London

29 Society Meeting and AGM, London (483)

January 2020

15 South West & South Wales Regional Meeting, Bristol

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

July

1-4 (Leaps through) Loops in Leeds, University of Leeds (482)

1-4 Dense Granular Flows IMA Conference, Cambridge (481)

1-5 LMS Research School: *Random Structures: from the Discrete to the Continuous,* Bath (482)

3-5 Current Status and Key Questions in Landscape Decision Making, INI, Cambridge (481)

8-9 UK Conference on Boundary Integral Methods, Oxford Brookes University (482)

8-12 LMS Research School, *Mathematics of Climate,* Reading

8-12 Measurability, Ergodic Theory and Combinatorics, University of Warwick (479)

8-12 Geometry, Compatibility and Structure Preservation in Computational Differential Equations, INI, Cambridge (481)

15-19 Computability in Europe 2019, Durham University (482)

22-26 Thermodynamic Formalism: Ergodic Theory and Geometry, University of Warwick (482)

22-26 Postgraduate Group Theory Conference 2019, University of Birmingham (481)

28-3 Aug International Mathematics Competition for University Students, Blagoevgrad, Bulgaria (481)

29-2 Aug British Combinatorial Conference 2019, University of Birmingham (479)

31-2 Aug Progress on Novel Mathematics and Statistics for Landscape Decisions, INI, Cambridge (481)

August

- 12-16 Factorisation of Matrix Functions, INI, Cambridge (481)
- 21-23 Algebraic Spline Geometry Meeting, Swansea University (483)
- 26-29 Caucasian Mathematics Conference, Rostov-on-Don, Russia

September

- 2 Applications of Mathematics in Ecology, University of Kent (483)
- 2 One Day Function Theory Meeting, De Morgan House, London (483)
- 4-6 Inverse Problems from Theory to Application, University College London (483)
- 5-6 Recent Trends in Stochastic Analysis and Partial Differential Equations, University of Chester (483)
- 5-6 Water Waves — Mathematical Theory & Applications, University of Plymouth (483)
- 5-7 British Logic Colloquium 2019, University of Oxford (483)
- 6-7 LMS Prospects in Mathematics Meeting, Lancaster (483)
- 8-11 Proof Society Summer School 2019, Swansea University (483)
- 9-11 Curve Counting Theories and Related Algebraic Structures, University of Leeds (483)
- 9-13 Graph Complexes in Algebraic Geometry and Topology, University of Manchester (482)
- 9-13 The Geometry of Derived Categories, University of Liverpool (483)
- 9-13 The Complex Analysis Toolbox: New Techniques and Perspectives, INI, Cambridge (482)
- 11 LMS Midlands Regional Meeting, Nottingham (483)
- 11-13 Proof Society Workshop 2019, Swansea University (483)
- 11-13 Groups and Representation Theory, Conference in Memory of Kay Magaard, University of Warwick (483)
- 12-13 Flood Risk Conference, Swansea University Bay Campus, Wales (483)
- 16 Random Matrices and Applications, University of Sussex (483)
- 16 Functor Categories for Groups, University of Lincoln (483)
- 16 Words in Finite and Profinite Groups, University of Lincoln
- 16-18 British Topology Meeting, Warwick (483)
- 17 Branching Processes and their Applications, University of Sussex (483)
- 18 Randomness, Symmetry and Free Probability, University of Sussex (483)
- 18-19 Philip Maini's 60th Birthday Workshop, Mathematical Institute, Oxford (483)
- 18-20 Southampton–Bielefeld Geometric Group Theory Meeting, University of Southampton (483)
- 18-21 Lattice Polytopes, with a View towards Geometry and Applications, ICMS, University of Edinburgh (483)
- 19 LMS Popular Lectures, University of Birmingham (483)
- 29-4 Oct Clay Research Conference and Workshops, Mathematical Institute, Oxford (482)
- 30-4 Oct Structure Preservation and General Relativity, INI, Cambridge (482)

October

- 10 Dependence Modeling and its Applications in Insurance and Finance, Heriot-Watt University (483)
- 7-11 Dirac Operators in Differential Geometry and Global Analysis, Bedlewo, Poland (482)
- 19 DeMorgan@21, De Morgan House, London (483)
- 28-1 Nov Complex Analysis in Mathematical Physics and Applications, INI, Cambridge (483)

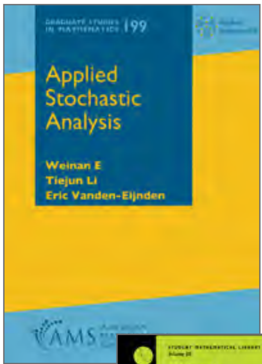
November

- 21 Joint LMS Meeting with the Institute of Mathematics and its Applications, Reading
- 29 Graduate Student Meeting, London
- 29 Society Meeting and AGM, London

December

- 6 PDE Models for Cancer Invasion, Queen's University Belfast
- 9-13 Computational Complex Analysis, INI, Cambridge

AMERICAN MATHEMATICAL SOCIETY



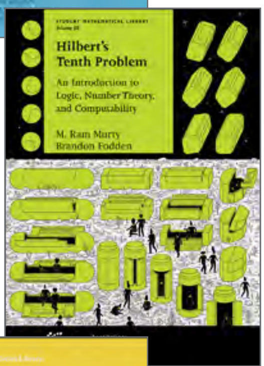
APPLIED STOCHASTIC ANALYSIS

Weinan E, Princeton University, Tiejun Li, Peking University & Eric Vanden-Eijnden, Courant Institute of Mathematical Sciences

Presents the basic mathematical foundations of stochastic analysis (probability theory and stochastic processes) as well as some important practical tools and applications. This book strikes a nice balance between mathematical formalism and intuitive arguments, a style that is most suited for applied mathematicians.

Graduate Studies in Mathematics, Vol. 199

Jun 2019 305pp 9781470449339 Hardback £81.95



HILBERT'S TENTH PROBLEM

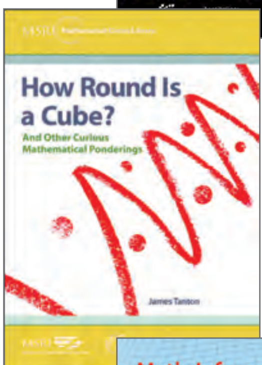
An Introduction to Logic, Number Theory, and Computability

M. Ram Murty, Queen's University & Brandon Fodden, Carleton University

Hilbert's tenth problem is one of 23 problems proposed by David Hilbert in 1900. It asked for a general algorithm to determine if a given Diophantine equation has a solution in integers. It was finally resolved by Julia Robinson, Martin Davis, Hilary Putnam, and finally Yuri Matiyasevich in 1970. This book is an exposition of their achievement.

Student Mathematical Library, Vol. 88

Jun 2019 242pp 9781470443993 Paperback £52.95



HOW ROUND IS A CUBE?

And Other Curious Mathematical Ponderings

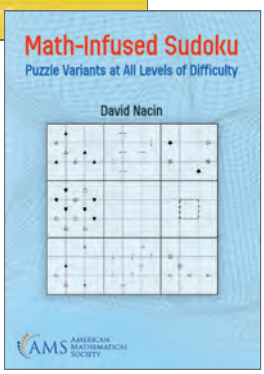
James Tanton, Mathematical Association of America

A collection of 34 curiosities, each a quirky and delightful gem of mathematics and each a shining example of the joy and surprise that mathematics can bring. Intended for the general math enthusiast, each essay begins with an intriguing puzzle, which either springboards into or unravels to become a wondrous piece of thinking. The essays are self-contained and rely only on tools from high-school mathematics (with only a few pieces that ever-so-briefly brush up against high-school calculus).

MSRI Mathematical Circles Library, Vol. 23

A co-publication of the AMS and the Mathematical Sciences Research Institute

Aug 2019 262pp 9781470451158 Paperback £24.50



MATH-INFUSED SUDOKU

Puzzle Variants at All Levels of Difficulty

David Nacin, William Paterson University

Presents nine variations on a classic and beloved puzzle type. Building upon the rules of Sudoku, the puzzles in this volume introduce new challenges by adding clues involving sums, differences, means, divisibility, and more. Each of the first eight chapters presents a rule system followed by a series of puzzles that progress in difficulty from easy to hard, allowing readers to develop and hone skills in logical problem solving, pattern discernment, and strategy building. In the final chapter, the eight puzzle variants are combined into a single complex puzzle type that puts all of the reader's new skills into play.

Jul 2019 116pp 9781470450908 Paperback £24.50

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